G W H Larkby
Dangur Deft.

ENGINEERING TRAINING SUPPLEMENT

No. 4

Second Issue

SOME FUNDAMENTAL PROBLEMS IN RADIO ENGINEERING

BRITISH BROADCASTING CORPORATION

ENGINEERING TRAINING SUPPLEMENT

No. 4

SOME FUNDAMENTAL PROBLEMS IN RADIO ENGINEERING

1. INDUCED E.M.F. IN AN INDUCTANCE
2. APPLICATIONS OF j NOTATION
3. VECTOR DIAGRAMS FOR TRANSFORMERS
4. SHORT-CIRCUITED TURNS IN COILS
5. HARMONIC DISTORTION IN VARIABLE-MU VALVES
6. CLASS-A AND CLASS-B OPERATION IN A.F. AMPLIFIERS
7. OUTPUT IMPEDANCE OF VALVE AMPLIFIERS WITH COMBINED CURRENT AND VOLTAGE FEEDBACK
8. DISTORTION IN DIODE DETECTORS DUE TO A.F. COUPLING CIRCUITS

EDITED and ISSUED BY
ENGINEERING TRAINING DEPARTMENT
B B C
February 1948

PREFACE

With the introduction of written Grade " D " examinations the need for information on fundamental as well as specialist theory becomes more urgent, and this training supplement has been planned as the first of a series to cater for the requirements of Grade " D " candidates.

The supplement is based on technical bulletins which have been issued to Instructors in the BBC Engineering Training Department during the past two years. Many of these bulletins, written by the Head of the Engineering Training Department, covered fundamental problems in Radio Engineering of importance to all those engaged in the technical activities associated with broadcasting.

The subjects under examination were often suggested by queries raised during discussions with Instructors, and each problem is complete in itself and not necessarily inter-related with its neighbours.

SOME FUNDAMENTAL PROBLEMS IN RADIO ENGINEERING

INTRODUCTION

Eight fundamental problems are examined in this supplement, which covers such subjects as induced e.m.f., *j* notation, transformer action, amplification, bridge negative feedback and detection. Confusion seems to exist concerning induced e.m.f. in a coil and the voltage appearing across the terminals of the same coil. This and the significance of the term " back " e.m.f. are considered in the opening problem and reference is made to the possibility of calling the voltage across a resistance a " back voltage." Considerable importance is attached to the second problem concerning the application of " *j* " notation, and attention is specially drawn to the usefulness of admittance in solving radio-frequency circuit calculations. Whilst *j* notation can prove a most valuable tool in the hands of the radio engineer, he cannot afford to neglect the vector representation of his results, and in the section on transformers the method of attack is by *j* notation but the result is interpreted in vector form. Problem 4 explains why turns of coils may be short-circuited at radio frequencies but must not be short-circuited at audio and power frequencies.

The next three problems are concerned with amplification ; the subject of one is harmonic distortion in variable-mu valves and it is shown why a single valve can be used for r.f. but not for a.f. operation : another indicates how a change of load impedance is caused by adding a second valve in push-pull to a Class-A output stage : the last shows the method of calculating amplifier output impedance when combined current and voltage feedback is employed as in the D/11 amplifier.

A simple explanation of modulation peak clipping in a detector due to an a.c./d.c. load resistance ratio less than unity completes the supplement.

1. INDUCED E.M.F. IN AN INDUCTANCE

Before starting a general discussion on induced e.m.f. it is important to differentiate between the voltage produced across a resistance by a current flowing through it, and the e.m.f. induced in a coil as the result of a current through the coil setting up an external magnetic field. In the first instance energy is continuously dissipated in the resistance, and the energy and voltage disappear as soon as the driving e.m.f. is withdrawn. In the second case energy is stored in the magnetic field set up round the coil, and it can be, and is, returned to the circuit when the driving e.m.f. decreases or is withdrawn. This induced e.m.f. exists only while the current is changing in value and always has such a polarity that it opposes the current change, i.e., when the current is increasing in value the induced e.m.f. acts against it and when it is decreasing in value the e.m.f. acts with it.

Fig. 1 illustrates a series circuit of R and L connected through a switch S to a resistanceless battery providing an e.m.f. of E volts.

At the instant when switch S is closed to position 1 the battery e.m.f. appears instantaneously across the inductance L, and an e.m.f. is generated

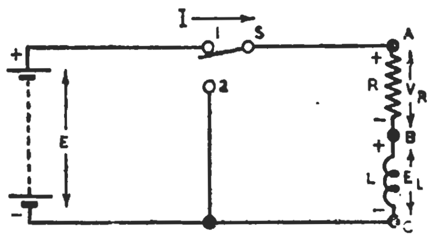


Fig. 1

in the coil to oppose the battery e.m.f., having a polarity making B positive with respect to C. The current round the circuit gradually builds up exponentially to a steady state value of E/R; the voltage across L, which is the induced e.m.f. in L, falls eventually to zero whilst that across R rises (A is, of course, positive with respect to B) to a steady state value of E volts.

If the switch S is moved to position 2 after the steady state has been reached, the magnetic field in L begins to collapse and induces an e.m.f. in the coil which tends to maintain the current in the circuit. The actual magnitude of the induced e.m.f. depends on the rapidity of breaking current if the changeover from position 1 to 2 is not instantaneous, and it has a polarity opposite to that originally produced when the driving e.m.f. is applied.

In order to emphasise the polarity change in the induced e.m.f., Figs. 2 and 3 are shown with a battery replacing the coil. The battery analogy must

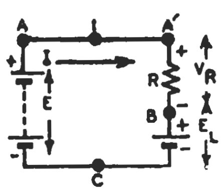


Fig. 2

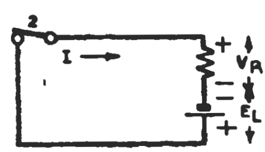


Fig. 3

not be pushed too far because many students imagine a battery as having an e.m.f. fixed in relation to time; in actual fact the induced e.m.f. only manifests itself when the current is changing, being zero when the latter is constant. The only excuse for using the battery illustration is that it indicates quite clearly the polarity of the induced e.m.f. and its reversal when the current in the circuit is reduced.

The actual time variation of voltage across the resistance, and of induced e.m.f. in the coil is shown in Fig. 4 for both positions of switch S. The full line curves for position 2 of the switch assume that the changeover from 1 to 2 is instantaneous so that the induced e.m.f. is equal to the original battery e.m.f. E, but is of opposite polarity to that of the induced e.m.f. when in position 1. In practice the changeover is not instantaneous and there is generally a time interval between the break of 1 and the make of 2. At the beginning of the break period the resistance in the circuit is infinite and a theoretically infinite e.m.f. appears across the coil in trying to maintain the current at its original value of $I = E/R$; actually, the e.m.f. never reaches an infinite value though it may be very large. It generally causes the air between the break points to ionise and produce a flashover; this immediately inserts a finite resistance in the circuit and limits the e.m.f.

The e.m.f. for the non-instantaneous make and break is shown by the dotted curve (position 2) in Fig. 4, which " rises " to a much higher negative

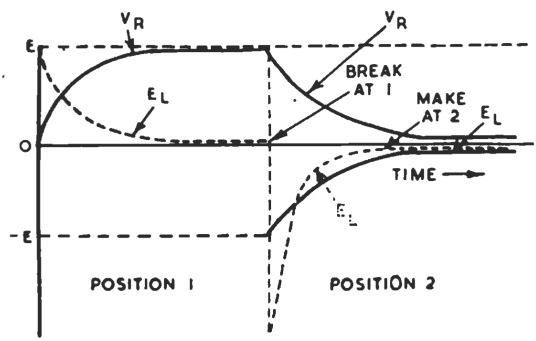


Fig. 4

value than the instantaneous break, but falls much more rapidly while the flashover lasts and then at the same rate from a lower negative value when position 2 is made. The time constant for the flashover is $\frac{L}{R+R_b}$, and for the make (position 2) is L/R, where R_b is the resistance of the arc of the flashover.

The induced e.m.f. in the coil due to a changing current in it is generally written as

$$E_L = -L\frac{di}{dt} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The necessity for this will be apparent from the statement made above, viz., that the induced e.m.f. has a polarity such as to oppose the current change producing it. It also follows from Kirchhoff's second law which states that "in any closed circuit the algebraic sum of the voltages in the various parts of the circuit is equal to the algebraic sum of the e.m.f.s acting round the circuit." Applying this law to the circuit of Fig. 1, we should write

$$E + E_L = E - L\frac{di}{dt}$$

$$= V_{AC} + V_{CB} = V_R = iR \qquad \ldots \quad \ldots \quad \ldots \quad (2a)$$

where V_{AC} = the voltage across AC in the direction A to C

and V_{CB} = the voltage across CB in the direction C to B.

Expression (1) may be rewritten

$$E_L = -L\frac{di}{dt} = V_{CB} = -V_{BC}$$

thus the voltage across BC in the direction B to C is

$$V_{BC} = L\frac{di}{dt}$$

Replacing E_L in (2a) by $-V_{BC}$

$$E - V_{BC} = V_R \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2b)$$

The sign attached to V_{BC} in expression (2b) emphasises the "counter" character of the induced e.m.f. by indicating that it acts against the driving e.m.f., and it explains why the term "back" e.m.f is often applied to an induced e.m.f.

If it is desired the induced e.m.f. can be associated with the voltage side of equation 2a so that

$$E = -E_L + V_R = L\frac{di}{dt} + V_R$$

$$= V_{BC} + V_{AB} \qquad \ldots \quad \ldots \quad \ldots \quad (3)$$

There is no fundamental difference between (2a) and (3), but (3) is the more useful expression when solving a.c. problems dealing with circuits containing R and L.

It is necessary now to consider the significance of expressions 2a and 3 in the vectorial representation of current and voltage in a circuit of L and R, to which is applied a sinusoidal a.c. driving e.m.f. (Fig. 5). Expression 2a becomes

$$\dot{E} + \dot{E}_L = \dot{V}_R \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

The dot over the e.m.f.s and voltages indicates that they are vectors and

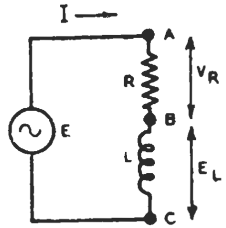


Fig. 5

not, therefore, necessarily in the same straight line. If the instantaneous current is represented by

$$i = \hat{I} \sin pt$$

where $\hat{I}$ is the peak value of the current, and $p = 2\pi f$;
the instantaneous voltage across the resistance is

$$v_R = iR = \hat{I}R \sin pt$$

and is in phase with the current.

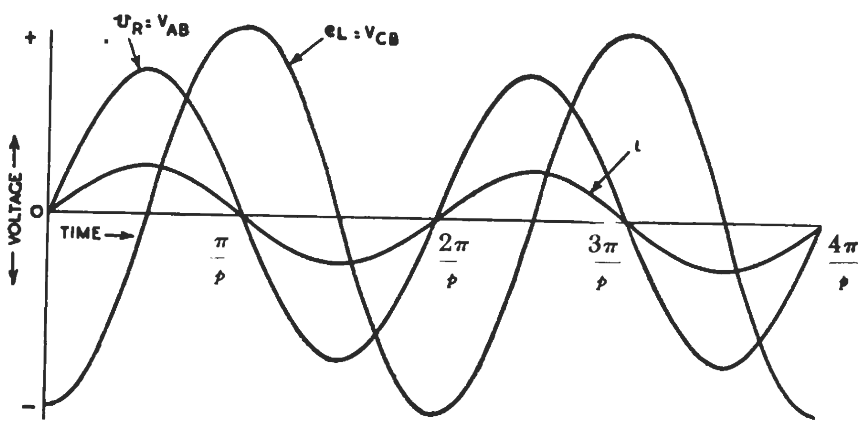


Fig. 6*a*

The instantaneous induced e.m.f. is

$$e_L = -L\frac{di}{dt} = -Lp\,\hat{I}\cos pt = Lp\,\hat{I}\,(-\cos pt)$$

and it lags 90° behind the instantaneous current.

The waveform and r.m.s. vector representations of (4) are therefore as shown in Figs. 6a and 6b, where the voltage across the resistance V_R is the voltage

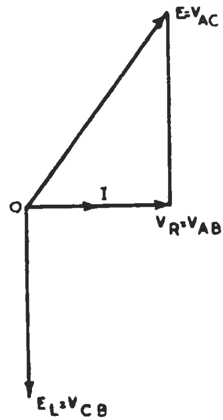


Fig. 6b

referred to the direction A to B, and the induced e.m.f. is the voltage across the coil in the direction C to B. In most radio problems the voltage across L and R is the important parameter, and in these circumstances the voltage appearing across L is required in the direction B to C. This gives rise to the vector representation of Fig. 7, where V_{AB} and V_{BC} are the components of the driving e.m.f. appearing across R and L respectively.
$V_{BC} = -V_{CB} = -E_L$ is the conventional inductive voltage vector leading upon the current I by 90°.

Some authorities* carry the argument of back e.m.f. further by suggesting that the voltage across a resistance is a " back " voltage in the sense that it opposes the flow of current in a resistance, and though this is a rather heterodox view it cannot be said to be fundamentally wrong. In effect, these authorities are writing expression (4) as

$$\dot{E} + \dot{E}_L - \dot{V}_R = 0 = \dot{E} - \dot{V}_{BC} - \dot{V}_{AB} = \dot{E} + \dot{V}_{CB} + \dot{V}_{BA}$$

and the vector diagram is that of Fig. 8.

---

* F. M. Colebrook. "*Basic Mathematics for Radio Students.*" Iliffe.

Summarising, Fig. 7 is the utilitarian representation of voltage and current vectors, most helpful to radio engineering students ; Fig. 6b illus-

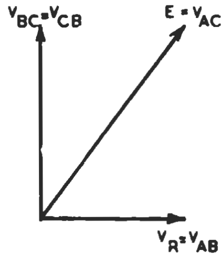


Fig. 7

trates the orthodox convention with the induced e.m.f. vector shown as a true " back " e.m.f., and is halfway towards the heterodox and yet perhaps more logical representation of Fig. 8. The student must realise that all the

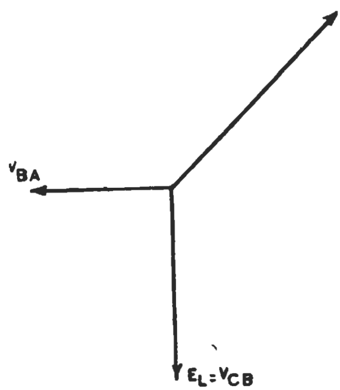


Fig. 8

diagrams are slightly different viewpoints of the same thing, and he would be wise to make no attempt (as is sometimes done) to combine Figs. 6b and 7, showing the voltage vector V_{BC} on the same diagram as E_L as if it possessed a separate identity.

2. APPLICATIONS OF j NOTATION

The importance of j notation for solving circuit problems in radio engineering is becoming generally recognised, and the fundamental principles involved in the operator " j " are included in many text books and in TT.5. It is unnecessary therefore to consider these and in this section only applications are examined.

Impedance, Resistance and Reactance

The three basic forms of the series circuit shown in Figs. 9a, 9b and 9c are written in j form as

$$Z = R + j\omega L \qquad \text{(5a)}$$

$$Z = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C} \qquad \text{(5b)}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \qquad \text{(5c)}$$

where $\omega = 2\pi$ x frequency.

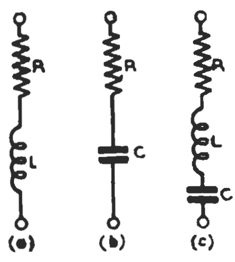


Fig. 9

The general form of the impedance expression is

$$Z = R + jX \qquad \text{(6)}$$

where X is the reactance, i.e., ωL if it is inductive and $-\frac{1}{\omega C}$ if it is capacitive.

Hence, if analysis of a complicated circuit results in an impedance expression having a positive sign associated with the j term, the reactance is inductive, or equivalently inductive. Alternatively, a negative sign associated with the j term implies a capacitive reactance. The qualification " equivalently " is employed in the above sentence to indicate that the inductive reactance resulting from the analysis may be a complicated function of frequency (containing powers of frequency greater than unity) and not of the form

$2\pi fL$. An example of an equivalent capacitive reactance is provided by the term

$$-j\omega L_2 \cdot \frac{\omega^2 M^2}{R^2 + \omega^2 L^2_2}$$

in section 3. If so desired, this term could be called a negative inductive reactance, since the reactance clearly tends to increase with increase of frequency, but in the writer's opinion it is preferable to call it an equivalent capacitive reactance.

Since the use of mathematics is merely a means to an end as far as the engineer is concerned, a mathematically expressed result is of no value unless it can be readily interpreted in a practical form. To illustrate the interpretation of j forms of the impedance vector, the following two expressions

$$Z_1 = 100 + j1000$$

and

$$Z_2 = 200 - j300$$

will be considered.

The practical equivalent of Z_1 is a resistance of 100 ohms in series with an inductive reactance of 1,000 ohms, and that of Z_2 is a resistance of 200 ohms in series with a capacitive reactance of 300 ohms. Figs. 10a and 10b

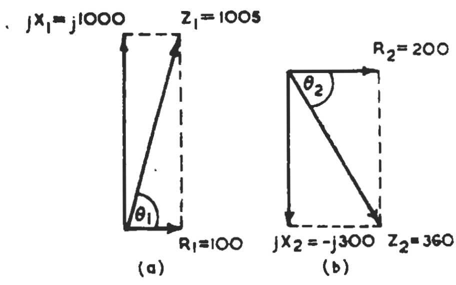


Fig. 10a & b

give the vectorial representations and it will be noted that the total impedance or modulus of Z_1, designated by $|Z_1|$ is

$$\sqrt{100^2 + 1000^2} = \sqrt{1,010,000} = 1005\ \Omega$$

The angle by which Z_1 leads upon R_1 is $\theta_1 = \tan^{-1}\frac{X_1}{R_1} = \tan^{-1}\frac{1000}{100} = 84° \ 18''$, and for this circuit the applied voltage will lead the current by this angle, $\theta_1 = 84° 18''$.

The modulus of Z_2 is $\sqrt{200^2 + 300^2} = \sqrt{130{,}000} = 360\,\Omega$ and Z_2 itself lags behind R_2 by an angle $\theta_2 = \tan^{-1} - \frac{300}{200} = -\ 56° 19''$. For this circuit the current will lead an applied voltage by an angle of 56° 19''.

The value of the modulus of impedance is seen to be independent of the sign associated with the j term, being always

$$|Z| = \sqrt{R^2 + X^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

This is to be expected since it is stated above that the sign is really associated with the reactance X, and where it is capacitive $|Z|$ may be written $\sqrt{R^2 + (-X_c)^2} = \sqrt{R^2 + X^2_c}$. The sign itself merely indicates whether the applied voltage leads or lags upon the current, a positive sign meaning a voltage lead and a negative a voltage lag. No further information of the practical form of Z_1 and Z_2 can be obtained unless the frequency is specified. If $f = 50$ c/s the inductive reactance of 1,000 ohms can be specified as that produced by an inductance of $L = \frac{1000}{2\pi f} = 3.185$ henry, and the capacitive reactance of 300 ohms as that produced by a capacitance $C = \frac{10^6}{2\pi f\, 300} = 10.6$ microfarad. The two practical circuits corresponding to the Z_1 and Z_2 j forms are as shown in Figs. 10c and 10d.

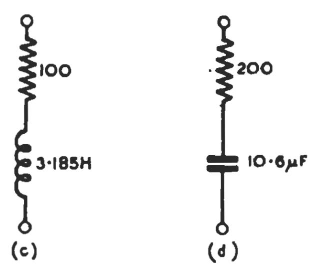


Fig. 10c & d

One further point needs to be stressed : impedance is essentially a series circuit parameter, and the total applied voltage and its resistive and reactive components are obtained by multiplying Z, R and X by the current I.

Thus

$$E \text{ applied} = E_R + iE_X$$

or

$$IZ = IR + jIX$$

and if a current of one ampere is flowing through the circuit Z_1

$$E \text{ applied} = 1005 \text{ volts}, \; E_R = 100 \text{ and } E_X = 1{,}000 \text{ volts}.$$

Admittance, Conductance and Susceptance

For the solution of parallel circuit problems—these tend to predominate in radio engineering—admittance, Y, the reciprocal of impedance, is a more useful parameter.

Since $Z = E/I$, $Y = I/E$, and multiplication of Y by the applied voltage E gives the total current in the circuit. The units for Y are mhos, and practical values are usually less than unity ; for example, an impedance of 100 ohms is equivalent to an admittance of $1/100 = 0.01$ mhos.

Just as Z has two components, so has Y : conductance, G, and susceptance, B, measured also in mhos and numerically equivalent to the reciprocal of parallel resistance and reactance. When multiplied by the applied voltage the two components give the conductance and susceptance components of the total current.

The general form of the admittance expression is

$$Y = G + jB \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

and the physical interpretation of this is a circuit consisting of a resistance, $1/G$ ohms, in parallel with a reactance, $-1/B$ ohms, as shown in Fig. 11.

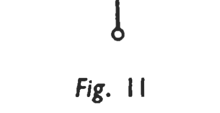


Fig. 11

The negative sign before the reciprocal of susceptance means that the sign of the equivalent parallel reactance is opposite to that of the susceptance, i.e., a capacitive reactance is equivalent to a positive susceptance and conversely an inductive reactance is equivalent to a negative susceptance. This is to be expected, since in a series circuit the reference vector is current and the total voltage is analysed into resistive and reactive components. An inductive reactive voltage leads upon the reference current vector, thus giving a positive sign to the reactance. In the parallel circuit the reference vector is voltage and the total current is analysed into conductive and susceptive components. An inductive susceptive current lags behind the reference voltage vector and this gives a negative sign to the inductive susceptance. Thus the sign attached to susceptance indicates whether the current leads or lags upon the voltage ; a positive sign means a current lead and a negative sign a current lag.

To illustrate this feature, an inductance L or reactance X_L is analysed vectorially as part of a series circuit in Fig. 12a and as part of a parallel circuit in Fig. 12b.

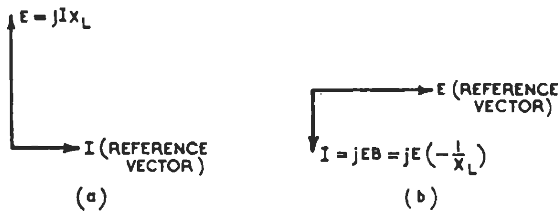


Fig. 12

Typical j forms of admittance are as follow :

$$Y_1 = 0.001 + j0.01$$
$$Y_2 = 0.002 - j0.02$$

and their practical equivalents are, for Y_1, a circuit consisting of a resistance

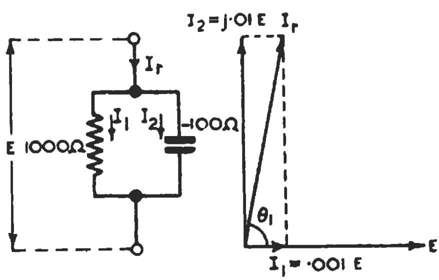


Fig. 13*a*

of 1,000 ohms (1/0.001) in parallel with a capacitive reactance of 100 ohms (1/0.01), and for Y_2 a resistance of 500 ohms in parallel with an inductive reactance of 50 ohms. These circuits and their vectorial representations are shown in Figs. 13a and 13b.

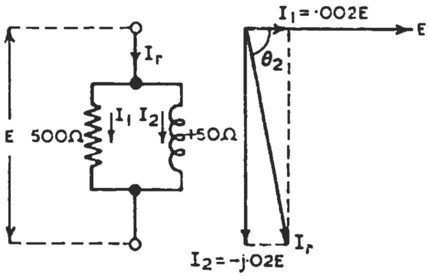


Fig. 13b

The total admittance or modulus of the admittance is

$$|Y| = \sqrt{G^2 + B^2}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

hence $$|Y_1| = \sqrt{(0.001)^2 + (0.01)^2} = 0.01005 \text{ mhos}$$

and $$|Y_2| = \sqrt{(0.002)^2 + (0.02)^2} = 0.0201 \text{ mhos}$$

If the applied voltage across Y_1 and Y_2 is 1,000 volts, the resistive, reactive and total currents are 1, 10 (capacitive, leading), and 10.05 amps. respectively, and 2, 20 (inductive, lagging) and 20.1 amps. respectively. The total current 10.05 amps. for Y_1 leads upon the voltage by an angle $\theta_1 = \tan^{-1} B/G = \tan^{-1} 0.01/0.001 = \tan^{-1} 10 = 84° 18'$, and the total current 20.1 amps. for Y_2 lags behind the voltage by an angle $\theta_2 = \tan^{-1} - 0.02/0.002 = - 84° 18'$.

Further information on Y_1 and Y_2 can be obtained if the frequency is specified. For example, when $f = 50$ c/s.

$$B_1 = 0.01 = 2\pi fC \text{ and } C = \frac{0.01 \times 10^6}{314} = 31.85\ \mu F$$

$$B_2 = -\ 0.02 = -\frac{1}{2\pi fL} \text{ and } L = 1/0.02 \times 314 = 0.159H$$

Conversion of Impedance to Admittance

As explained above, many r.f. problems require the solution of circuits into the admittance form, and in complicated analyses it may be necessary to convert a series circuit into its equivalent parallel form. In a series circuit of R and X, the impedance is

$$Z = R + jX$$

and the admittance is

$$Y = \frac{1}{Z} = \frac{1}{R + jX} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(10a)}$$

Rationalising 10a

$$Y = \frac{R - jX}{(R + jX)(R - jX)} = \frac{R - jX}{R^2 + X^2}$$

$$= \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = G + jB \ldots \quad \ldots \quad \ldots \text{(10b)}$$

from which $G = \frac{1}{R_p} = \frac{R}{R^2 + X^2}$ and $B = -\frac{1}{X_p} = \frac{-X}{R^2 + X^2}$ Hence the series circuit of R and X is identical with and may be replaced by a parallel circuit consisting of a resistance $(R^2 + X^2)/R$ in parallel with a reactance of $(R^2 + X^2)/X$ as shown in Fig. 14.

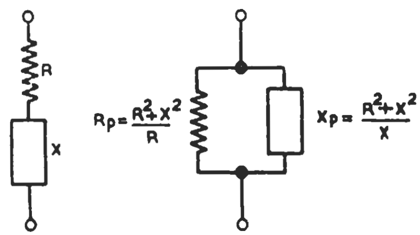


Fig. 14

To illustrate the advantages of such a conversion consider a series circuit of $R = 1{,}000$ ohms and $C = 5.05$ $\mu\mu$F placed across a circuit tuned to 1,000 kc/s by a capacitance of 200 $\mu\mu$F and having a resonant impedance of 100,000 ohms (Fig. 15a). The effect of adding this series circuit across the resonant circuit cannot be visualised, but if its parallel equivalent is found it becomes immediately obvious. (Fig. 15b.)

$$\text{At 1,000 kc/s } X_C = \frac{10^{12}}{6.28 \times 10^6 \times 5.05} = 31{,}500 \text{ ohms}$$

$$R_p = \frac{1}{G} = \frac{R^2 + X^2}{R} = \frac{1000^2 + 31500^2}{1000} \simeq 1{,}000{,}000 \text{ ohms}$$

$$X_p = \frac{R^2 + X^2}{X} = \frac{1000^2 + 31500^2}{-31500} \simeq -31{,}550 \text{ ohms}$$

$$X_p = \frac{-1}{\omega C} \simeq -31550 \text{ whence } C \simeq 5.04\ \mu\mu F$$

The two forms of the complete circuit are shown in Figs. 15a and 15b. The parallel form of circuit indicates that the original circuit is damped by an

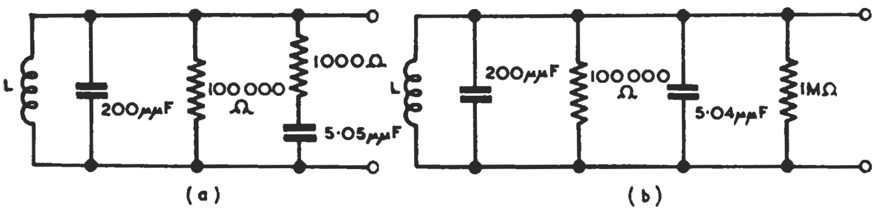


Fig. 15

additional 1,000,000 ohms, reducing its resonant impedance to 45,000 ohms, and it is distuned by the additional capacitance of 5.04 $\mu\mu F$, its new resonant frequency being approximately $1000 \times \sqrt{\frac{200}{205.04}} = 988$ kc/s.

Admittance in Valve Problems

Valve problems are generally analysed more readily by using the admittance concept, and this is particularly true of the variable-reactance valve, an example of which is given in Fig. 16. The anode-cathode circuit of

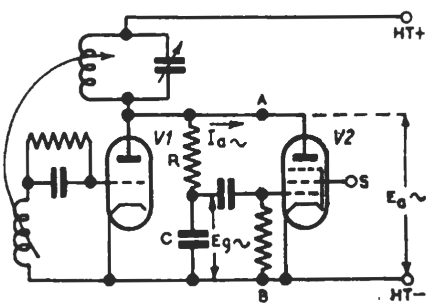


Fig. 16

the variable-reactance valve (V_2) is placed across the tuned circuit of the oscillator (V_1), and its grid r.f voltage is derived from a phase-shifting network R and C across the oscillator tuned circuit. This grid r.f. voltage E_g

is given approximately 90° phase shift and it produces a r.f. anode current which is in phase with E_g and therefore approximately 90° out-of-phase with the r.f. anode voltage. Such a condition is that found with a reactance, and if the bias on the control grid (or some other grid, if a multi-electrode valve) is varied the r.f. anode current r.m.s. value is varied, and the equivalent of a variable reactance is obtained.

$$Eg = \frac{E_a \times (-jX_c)}{R - jX_c} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$I_a = g_m Eg \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

if the valve is a tetrode and R_a is very large.

Replacing Eg in (12) by its value from (11)

$$I_a = \frac{g_m E_a \,.\, (-jX_c)}{R - jX_c}$$

The admittance looking in from the points AB towards V_2

$$Y_{AB} = \frac{I_a}{E_a} = \frac{-jg_m X_c}{R - jX_c}$$

$$\text{Rationalising } Y_{AB} = \frac{-jg_m X_c (R + jX_c)}{R^2 + X_c^2}$$

$$= \frac{-j^2 g_m X_c^2}{R^2 + X_c^2} - \frac{jg_m X_c R}{R^2 + X_c^2}$$

$$= \frac{g_m X_c^2}{R^2 + X_c^2} - \frac{j\, g_m X_c R}{R^2 + X_c^2} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Expression (13) is equivalent to a resistance of $(R^2+X_c^2)/(g_m X_c^2)$ in parallel with an inductive reactance of $(R^2+X_c^2)/(g_m X_c R)$. It is the second term which gives the variable reactance valve its characteristic and makes it a useful frequency modulator. The first term damps the tuned circuit and causes a variation in oscillation amplitude ; in practice special precautions are taken to reduce or cancel its effect.

The Parallel Tuned Circuit

The parallel tuned circuit of Fig. 17a illustrates a very common form of practical circuit, and its series or parallel equivalents at frequencies other than the resonant frequency are quite often required.

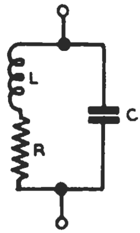
For the series equivalent

$$Z = \frac{(R + j\omega L)\left(\dfrac{-j}{\omega C}\right)}{R + j\omega L - \dfrac{j}{\omega C}}$$

Multiplying top and bottom by $\dfrac{\omega C}{-j}$

$$Z = \frac{R + j\omega L}{\dfrac{\omega CR}{-j} - \omega^2 LC + 1}$$

$$= \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}$$



(a)

Fig. 17a

Rationalising

$$Z = \frac{(R + j\omega L)\,[(1 - \omega^2 LC) - j\omega CR]}{(1 - \omega^2 LC)^2 + (\omega CR)^2}$$

$$= \frac{R(1 - \omega^2 LC) + \omega^2 LCR + j\omega L\,(1 - \omega^2 LC) - j\omega CR^2}{(1 - \omega^2 LC)^2 + (\omega CR)^2}$$

$$= \frac{R}{(1 - \omega^2 LC)^2 + (\omega CR)^2} + j\,\frac{\omega L(1 - \omega^2 LC) - \omega CR^2}{(1 - \omega^2 LC)^2 + (\omega CR)^2}$$

which is equivalent to a resistance, R_S, of $\dfrac{R}{(1 - \omega^2 LC)^2 + (\omega CR)^2}$

in series with a reactance, X_S, of $\dfrac{\omega\,[L\,(1 - \omega^2 LC) - CR^2]}{(1 - \omega^2 LC)^2 + (\omega CR)^2}$

These two components may be interpreted in a more suitable engineering form by noting that $X_L = \omega L$ and $X_C = 1/\omega C$.

Thus

$$R_S = \frac{R}{(1 - \omega^2 LC)^2 + (\omega CR)^2} = \frac{R}{\left(1 - \dfrac{X_L}{X_C}\right)^2 + \left(\dfrac{R}{X_C}\right)^2}$$

and $$X_S = \frac{\omega[L\,(1 - \omega^2 LC) - CR^2]}{(1 - \omega^2 LC)^2 + (\omega CR)^2} = \frac{\omega\left[L\left(1 - \dfrac{X_L}{X_C}\right) - CR^2\right]}{\left(1 - \dfrac{X_L}{X_C}\right)^2 + \left(\dfrac{R}{X_C}\right)^2}$$

The sign of R_S is unaffected by the relative magnitudes of X_L and X_C but that of X_S depends on whether $X_L >$ or $< X_C$.
The reactance X_S becomes zero at the resonant frequency, i.e., when

$$1 - \frac{X_L}{X_C} = \frac{CR^2}{L} = 1 - \omega_r^2 LC$$

Therefore the resonant frequency f_r is

$$f_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}}$$

$$= f_{ro}\sqrt{1 - \frac{CR^2}{L}}$$

where f_{ro} = the resonant frequency when $R = O$.
If the above value of f_r is inserted in the expression for R_S the well-known resonant condition $R_S = L/CR$ is obtained. This same value of resonant impedance is obtained from the initial expression for Z after assuming that R can be neglected in comparison with ωL and that $f_r = 1/2\pi\sqrt{LC}$.
For frequencies below resonance X_L/X_C is less than unity and X_S is positive,

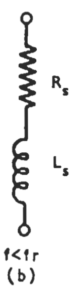


Fig. 17b

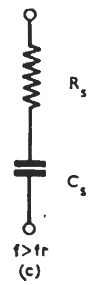


Fig. 17c

thus indicating that the parallel circuit appears as an inductance in series with a resistance. (See Fig. 17b.)

Conversely, above resonance X_S is negative and the circuit appears as a capacitance in series with a resistance. (See Fig. 17c.)

This result could have been deduced by considering which branch takes the larger current. For example, at frequencies below resonance the inductive branch has a lower impedance and therefore takes the larger current. The total current must therefore be preponderantly inductive, and the circuit appears as a resistance in series with an inductance.

For the parallel equivalent of Fig. 17a

$$Y = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{1 + j\omega C\,(R + j\omega L)}{R + j\omega L}$$

$$= \frac{(1 - \omega^2 LC + j\omega CR)}{R + j\omega L}$$

Rationalising

$$Y = \frac{(1 - \omega^2 LC + j\omega CR)\,(R - j\omega L)}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\,\frac{CR^2 - L(1 - \omega^2 LC)}{R^2 + \omega^2 L^2}$$

which is equivalent to a resistance $R_p = \dfrac{R^2 + \omega^2 L^2}{R}$ in parallel with a reactance of $X_p = \dfrac{R^2 + \omega^2 L^2}{\omega\,[L(1 - \omega^2 LC) - CR^2]}$

Alternative expressions are

$$R_p = \frac{R^2 + X_L{}^2}{R}$$

and

$$X_p = \frac{R^2 + X_L{}^2}{\omega\left[L\left(1 - \frac{X_L}{X_C}\right) - CR^2\right]}$$

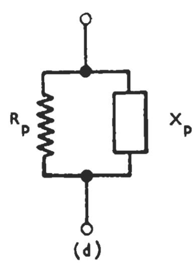


Fig. 17d

and the equivalent circuit is as shown in Fig. 17d.

The resonant frequency, obtained by making the susceptance component of Y equal to zero, is the same as for the series equivalent, and, as before, the reactance is positive or inductive at frequencies below resonance, and is negative or capacitive at frequencies above resonance.

3. VECTOR DIAGRAMS FOR TRANSFORMERS

The vector analysis of a transformer is fundamentally the vector analysis of circuits containing mutual inductance coupling, and the purpose of this section is to consider reflected impedance and the voltage and current relationships existing in such circuits. Consideration is given first to the simplest case of mutual inductance coupling between two sections of one coil, and later the theory is developed to cover the auto-transformer and the transformer with separate primary and secondary windings. The results are equally applicable to power and audio frequencies as to radio frequencies.

The Series-aiding Connection or Positive Mutual Inductance Coupling

The simplest example of mutual inductance coupling is supplied by two sections of a coil coupled together and series-connected in such a manner that their total inductance is greater than the sum of their separate inductances. Such coils are often termed series-aiding, because the two fluxes produced by the current flowing in the two coils have the same polarity.

Dependent upon the magnetic leakage, the two fluxes link to form a common flux embracing turns of both coils. The circuit, illustrated in Fig. 18a, is assumed, for convenience, to be resistanceless, and the driving e.m.f. is taken to be sinusoidal of r.m.s. value E volts. If it produces a current

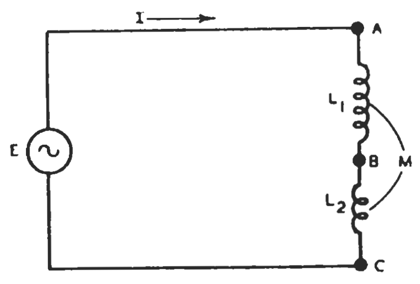


Fig. 18a

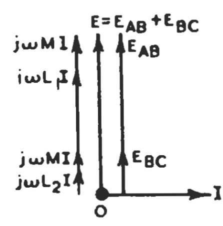


Fig. 18b

of r.m.s. value I amps. in the circuit, there are four e.m.f. components across the points AC ; two, due to the " back " e.m.f. of self induction, are of values $j\omega L_1I$ and $j\omega L_2I$*, and the other two, one produced in each coil due to current in the other, have equal values of $j\omega MI$. The vector representation is therefore as shown in Fig. 18b ; the voltage across coil 1 is $E_{AB} = j\omega(L_1 + M)I$, and that across coil 2 is $E_{BC} = j\omega(L_2 + M)I$. If the voltage across the second coil is considered in the direction C to B instead of B to

* Note that the operator j indicates that ωL_1I leads upon the current vector I by 90°. Conversely $-j$ indicates a lag by 90°.

C, the vector diagram is that of Fig. 18c, and it is shown later that this alternative is helpful in solving many transformer problems.

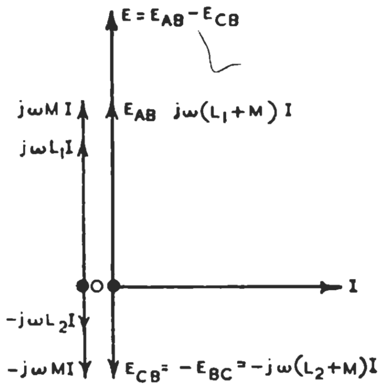


Fig. 18c

If the driving e.m.f. is now applied to the coil L_1 only, and the second coil is left on open circuit (see Fig. 19a), an e.m.f. will be induced in L_2 due

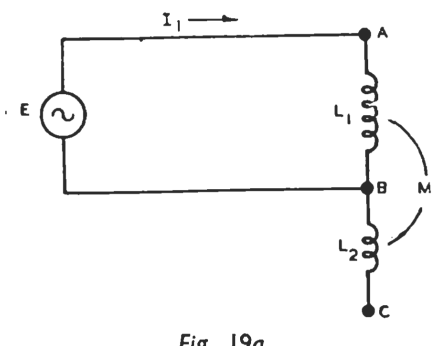


Fig. 19*a*

to the current change in L_1 and it will have a value $j\omega MI_1$ in the direction BC, or a value $-j\omega MI_1$ in the direction CB. There will not be, however, an e.m.f. induced in L_1 due to mutual coupling to L_2, since there is no current in L_2. The vector diagram and its alternative (similar to Fig. 18c) are shown in Figs. 19b and 19c. In this circuit we have the simplest prototype auto-transformer. Fig. 19a can be re-arranged as in Fig. 20 to give the simplest form of transformer with separate primary and secondary coils but with one primary and one secondary terminal connected. The turns of coil L_2 are shown reversed in the diagram so as to emphasise that the flux which would be produced by I_1 flowing in L_2 would be of the same polarity as that produced by I_1 flowing in L_1. It should be noted that the voltage developed across the secondary terminals, E_{CB}, is 180° out of phase with the driving

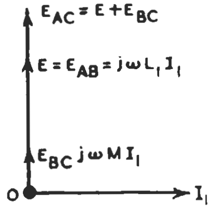


Fig. 19b.

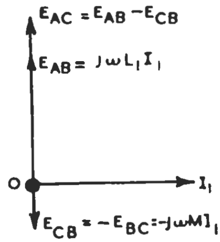


Fig. 19c

voltage applied to the primary terminals, i.e., Fig. 19c gives the vector representation.

If the terminals CB are short-circuited, a current I_2 is produced in L_2 (Fig. 21a) and this current induces in the primary coil L_1 an e.m.f., which is

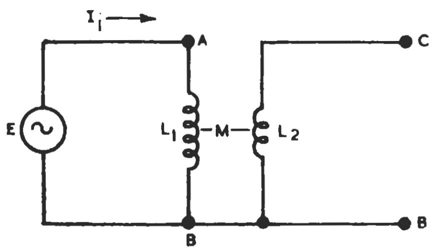


Fig. 20

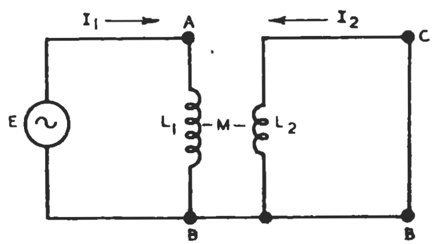


Fig. 21a

in a direction to add to the driving e.m.f., and so increase the current I_1 flowing in the primary. The e.m.f., induced in L_1 from L_2, causes a current in L_2 in the opposite direction to that which would result were the driving e.m.f. to be applied across AC as in Fig. 18a. Thus, in the vector diagram of Fig. 21b, I_2 is shown as a vector pointing in the opposite direction to I_1. Because of this reversed direction of secondary current the e.m.f. induced in L_1 from L_2 ($j\omega MI_2$) is opposite in direction to the e.m.f. of self induction and cancels part of it, so aiding the driving e.m.f. The effect on the primary is exactly the same as if the primary inductance has been reduced, for

$$E = I_1 j\omega L_1 + I_2 j\omega M \quad \ldots \quad .. \quad \ldots \quad (14)$$

$$- I_1 j\omega M = I_2 j\omega L_2 \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

so that
$$E = I_1\, j\omega L_1 - I_1\, j\omega M \,.\, \frac{M}{L_2}$$
$$= I_1\, j\omega \left[L_1 - \frac{M^2}{L_2} \right] \qquad \ldots \qquad \ldots \qquad \ldots \qquad (16)$$

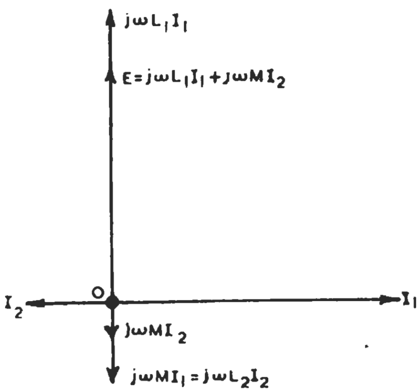


Fig. 21b

By noting that the coefficient of coupling $k = M/\sqrt{(L_1 L_2)}$, the effective primary inductance $L_{AB} = L_1(1 - k^2)$, and if $k = 1$, $L_{AB} = 0$, or a short-circuit of the secondary results in a short-circuit of the primary. If $k < 1$, the resulting inductance L_{AB} is the leakage inductance.

The next stage in the investigation is to place a resistance R across the secondary terminals as in Fig. 22a. Owing to the resistance the phase angle

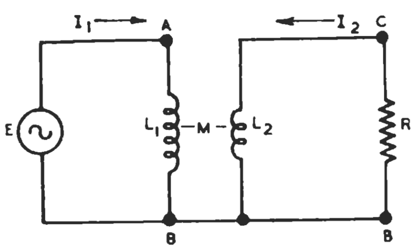


Fig. 22a

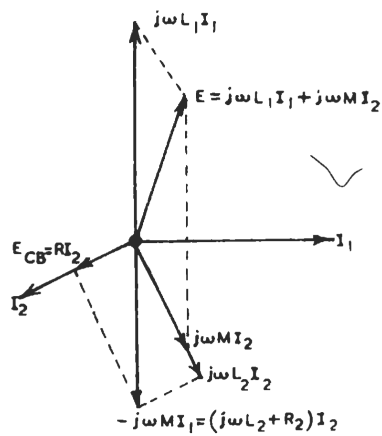


Fig. 22b

between the driving e.m.f. and primary current I_1, and that between the secondary induced e.m.f. $-j\omega MI_1$ and secondary current I_2 is no longer 90°, though the phase angle between the primary current I_1 and secondary induced e.m.f. is still 90°. The vector diagram is that of Fig. 22b, and it is seen that the effect of the e.m.f. induced into the primary by the secondary current is not only to reduce the effective primary inductance L_{AB} but also to insert a resistance component in series with it. It is interesting to note that this series resistance component is not a simple function of R but actually depends on ω, M, L_2 and R, and has a maximum value when $R = \omega L_2$. The effective primary inductance and resistance component can be calculated as follows :—

$$E = I_1 \, j\omega L + I_2 \, j\omega M \quad \ldots \quad (17)$$

$$-I_1 \, j\omega M = I_2 \, (j\omega L_2 + R) \quad \ldots \quad (18)$$

$$\therefore \quad E = I_1\left[j\omega L_1 + \frac{\omega^2 M^2}{R + j\,\omega L_2}\right] \quad \ldots \quad (19a)$$

$$= I_1 \left[j\omega L_1 + \frac{\omega^2 M^2 (R - j\omega L_2)}{R^2 + \omega^2 {L_2}^2}\right]$$

$$= I_1 \left[j\omega \left(L_1 - L_2 \,.\, \frac{\omega^2 M^2}{R^2 + \omega^2 {L_2}^2}\right) + R \, \frac{\omega^2 M^2}{R^2 + \omega^2 {L_2}^2}\right] \quad (19b)$$

$$\text{Thus } L_{AB} = L_1 - L_2 \,.\, \frac{\omega^2 M^2}{R^2 + \omega^2 {L_2}^2} \quad \ldots \quad (20)$$

$$\text{And} \quad R_{AB} = R \,.\, \frac{\omega^2 M^2}{R^2 + \omega^2 {L_2}^2} \quad \ldots \quad (21)$$

So far the analysis has been developed from two coils connected so that their mutual inductances add to their self inductances. A start could, however, have been made with two coils connected in " series-opposition " with their mutual inductances opposing the self inductances, and this problem will be considered next.

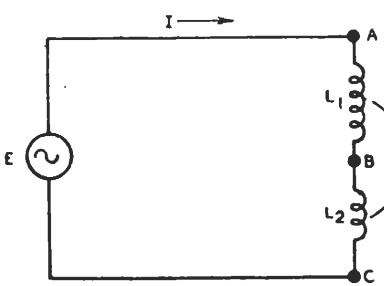


Fig. 23a

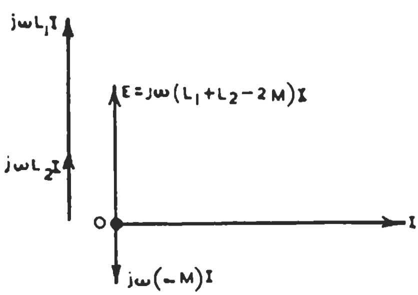


Fig. 23b

The Series-opposition Connection or Negative Mutual Inductance Coupling

In the series-opposition connection the sign of mutual inductance M is negative and the directions of all vectors containing the mutual inductance factor M have to be reversed. Vector diagrams corresponding to Figs. 18b and 19c are given in Figs. 23b and 24b, and the chief point of interest is that

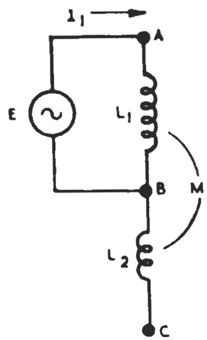


Fig. 24a

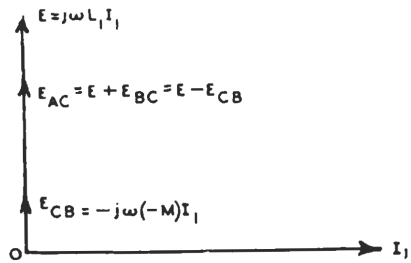


Fig. 24b

the induced voltage E_{CB} in the auto-transformer (Fig. 24b), is in phase with the driving e.m.f. and not 180° out-of-phase as for the first condition of mutual inductance.

When the secondary coil L_2 is short-circuited (Fig. 25a), the secondary current I_2, caused by the e.m.f. induced from L_1, is in the same direction as that which would result were the driving e.m.f. applied across the points AC.

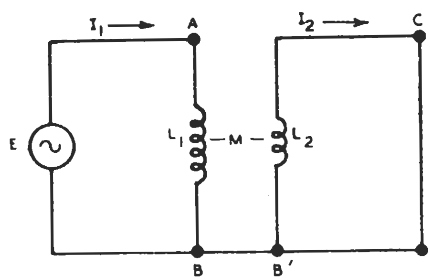


Fig. 25a

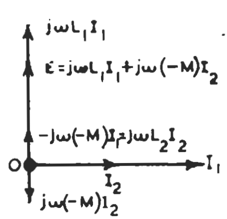


Fig. 25b

The current vector I_2 is therefore in phase with the current vector I_1, and the diagram is as shown in Fig. 25b, which corresponds with Fig. 21b. The expressions for voltage and current corresponding to (14) and (15) are modified to

$$E = I_1\, j\omega L_1 + I_2\, j\omega(-M) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

$$-I_1\, j\omega(-M) = I_2\, j\omega L \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

but the final result as far as the primary is concerned is unchanged because

$$E = I_1\, j\omega L_1 - I_1\, j\omega(-M)\,\frac{(-M)}{L_2}$$

$$= I_1\, j\omega\left[L_1 - \frac{M^2}{L_2}\right]$$

The same is true when the secondary is terminated by a resistance R, the vector diagram for which is that of Fig. 26.

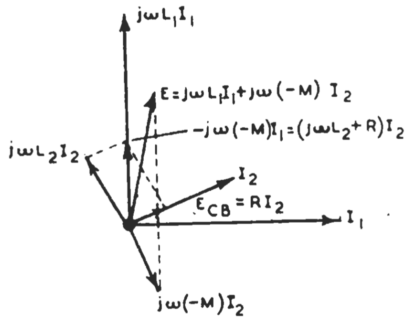


Fig. 26

Thus the sole effect of changing the sign of the mutual inductance between the coils is that the secondary terminal voltage is reversed in phase ; this is to be expected, since changing the sign of mutual inductance is accomplished either by reversing the secondary connections with respect to the primary, i.e., open-circuiting BB′ in Fig. 25a and joining C to B, or by reversing the direction of the primary or secondary turns. The primary current is unaffected by the sign of M and the effective primary inductance and resistance are unchanged. In fact it can be said that unless there is some form of coupling other than mutual inductance between the circuits, it does not matter what the sign of M is. Before considering the problem of double coupling it is worth while noting that equivalent T section circuits can be produced. The equivalent T section diagram for Fig. 22a is shown in Fig. 27. It can be seen that across the points AC of the series arm is the total inductance $L_1 + L_2 + 2M$, as would be expected, whilst the shunt arm is $-M$. As a check it may be noted that

$$(I_1 + I_2)\, j\omega(-M) = I_2\, j\omega(L_2 + M) + I_2 R$$

$$\text{or } I_1\, j\omega(-M) = -I_1\, j\omega M = I_2(j\omega L_2 + R)$$

which agrees with Fig. 22b, and also that the impedance looking in at the primary terminals AB is

$$j\omega(L_1 + M) + \frac{j\omega(-M) \,.\, (R + j\omega(L_2 + M))}{R + j\omega L_2}$$

$$= j\omega(L_1 + M) + j\omega(-M) + \frac{j\omega(-M) \,.\, j\omega M}{R + j\omega L_2}$$

$$= j\omega(L_1) + \frac{\omega^2 M^2}{R + j\omega L_2}$$

which is the same as expression (19a).

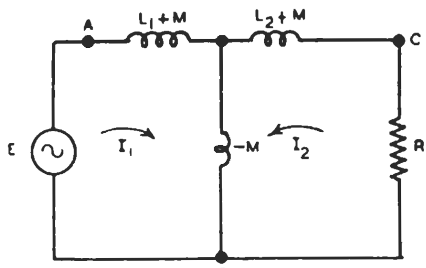


Fig. 27

The equivalent T section when M is reversed is as in Fig. 28; the shunt arm is now $+ M$. It is worth while warning students that there is some difference of opinion as to the sign convention for M. The orthodox view is that a positive sign should be given when M adds to total inductance giving the series arms $L_1 + M$ and $L_2 + M$, i.e., it is the series-aiding connection, and a negative sign is then given to the reversed or series-opposing connection

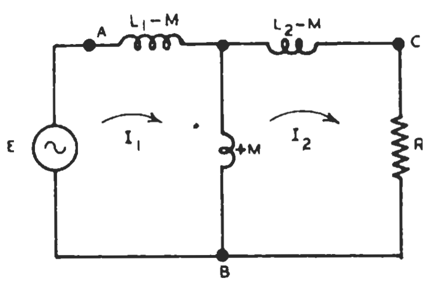


Fig. 28

when the series arms are $(L_1 - M)$ and $(L_2 - M)$. Some authorities (National Physical Laboratory Publications) adopt the opposite convention denoting the series-opposing connection as the positive value of M. This is justified on the grounds that the coupling arm (see Fig. 28) should give its sign to M. Adopting the orthodox convention, a positive value of M gives a secondary output voltage reversed in phase (not necessarily by 180°) upon the input e.m.f., and the negative sign produces a secondary voltage " in phase " (though not necessarily exactly at 0°) with the input e.m.f. Either convention if pursued logically will give the same result.

As far as the primary is concerned it has already been shown that the result is the same whatever sign M has, and its only effect is on the phase of the secondary output voltage relative to the input e.m.f. Except in problems involving feedback it does not matter whether M is positive or negative so long as there is no other form of coupling between the primary and secondary. In most practical cases there is coupling, intentional or accidental, in addition to mutual-inductance coupling. An example of intentional additional couplings can be found in certain types of r.f. band-pass filters, and accidental double-coupling is produced in an a.f. transformer which has capacitance between the primary and secondary windings. In all these cases the sign of M is most important and the transformer performance may be completely altered if the sign of M is changed, for example, by reversing primary or secondary connection.

Effect of the Sign of M in Double-coupled Circuits

As an illustration of the effect of the sign of M in an intentionally double-coupled transformer, consider the vectorial analysis of the circuit shown in Fig. 29, first with M positive (according to the orthodox view) and

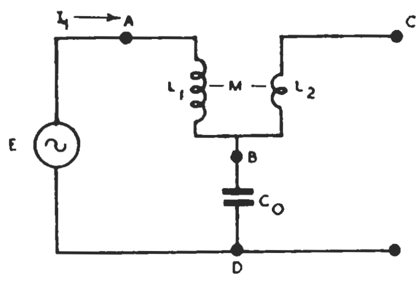


Fig. 29

then with M negative. To simplify the examination the secondary is assumed to be open-circuited. From Fig. 19c, the voltage E_{CB} induced in L_2 from L_1 is $-I_1 j\omega M$, and this is in phase with the voltage across the capacitor C_o, $-I_1 j/\omega C_o$, which lags by 90° behind the current vector I_1. Hence, as shown in Fig. 30, the coupling provided by C_o acts in the same direction as that due to $+M$, and increases the voltages injected into the secondary circuit. When M is negative, Fig. 24b shows that the induced voltage in L_2 is reversed, and the vector diagram is that of Fig. 31. The voltage induced in L_2 now opposes that due to C_o, and at one particular frequency the net voltage is zero. This occurs when

$$\omega_o M = \frac{1}{\omega_o C_o}$$

i.e.

$$f_o = \frac{1}{2\pi\sqrt{MC_o}} \quad \dots \quad \dots \quad \dots \quad \dots \quad (24)$$

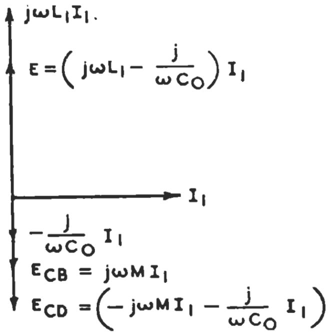


Fig. 30

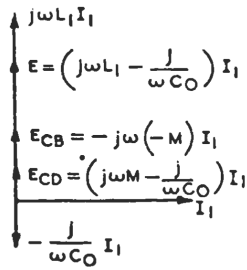


Fig. 31

Referring to the equivalent T-section network, of Fig. 32, this implies series resonance of the shunt arm. There is no possibility of series resonance when M is positive; because the shunt arm M is then negative, and

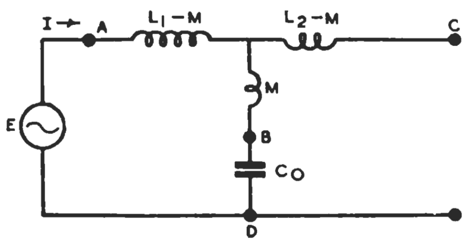


Fig. 32

$\omega(-M) - (1/\omega C_o)$ can never be zero. It is therefore clear that the sign of M is most important and that transformer performance will be quite different for negative from what it is for positive M.

An illustration of unintentional double-coupling provided by inter-winding capacitance is shown in Fig. 33. The vector analysis has the additional complication of current in the secondary coil L_2 due to the coupling capacitance. If M is positive and the reactance of C_o is much greater than that of L_2 this current is in the opposite direction to that in which it would flow were the driving e.m.f. to be connected across AC. The vector diagram

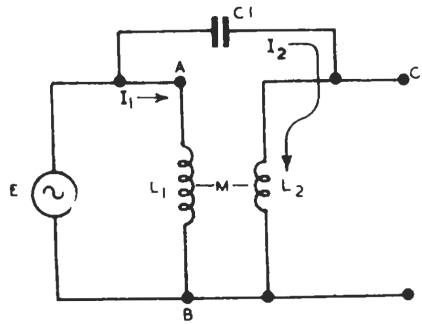


Fig. 33

is therefore as in Fig. 34. The voltage E_{CB} is the sum of that injected from L_1 $(- I_1 j\omega M)$ and that due to the current I_2 in L_2 and, as shown by the diagram, the two are additive. The effect of the coupling capacitance C_1 is

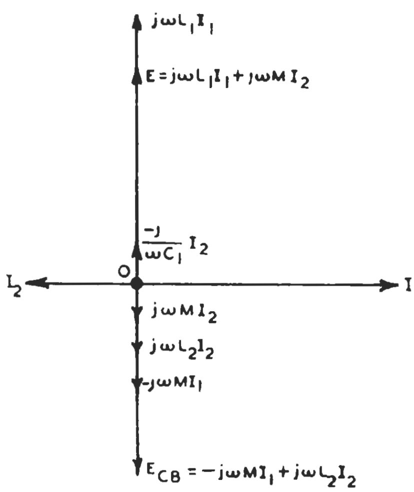


Fig. 34

therefore to increase the secondary voltage and is identical with that obtained by the series capacitor C_o. When M is negative the induced voltage in L_2

from L_1 reverses and opposes I_2 $j\omega L_2$ (Fig. 35) and at one particular frequency given by

$$f_1 = \frac{1}{2\pi\sqrt{\frac{(L_1L_2 - M^2)}{M}\,C_1}}$$

the two voltages cancel to produce zero secondary voltage. This is again similar to the condition when the series capacitor C_o is used.

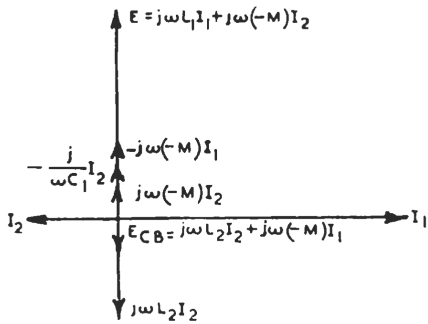


Fig. 35

A sufficiently detailed treatment has been given to allow an interested student to extend the examination to cover all practical cases, such as by the addition of resistance in series with L_1 and L_2 and the insertion of primary and secondary tuning capacitors to cover r.f. and i.f. transformers.

4. SHORT-CIRCUITED TURNS IN COILS

It is well known that turns in a coil forming part of a radio-frequency tuned circuit may be shorted in order to change the tuning inductance, but that short-circuited turns in a power or audio-frequency transformer can cause serious damage or completely ruin its performance.

The explanation for the different effects of a short-circuit is that, (1), the inductive reactance of the short-circuited turns of a r.f. coil is generally much greater than the resistance because of the higher operating frequency, (2), the coupling coefficient, $k=M/\sqrt{(L_1L_2)}$, is less, and (3), the ratio of the total to short-circuit inductance is much less than in the power transformer. The last two factors cause a short-circuit circulating current little different from that in the main section of the r.f. coil, whereas for the power or a.f. transformer the circulating current is usually very large compared with the current in the rest of the coil, and relatively very large power is dissipated in the short-circuited section.

To illustrate this point a possible circuit will be analysed and appropriate values inserted. Fig. 36 illustrates a coil having a short-circuited section,

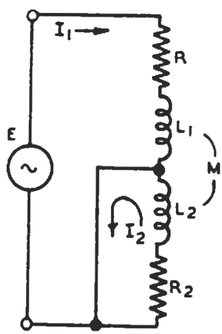


Fig. 36

and for convenience it is assumed that the short-circuit has zero resistance. L_2 and R_2 represent the inductance and resistance of the short-circuited section of the coil and L_1 and R_1 the inductance and resistance of the remainder of the coil. If I_1 is the current taken from the source of driving voltage E, and I_2 is the current circulating in the short-circuited section, the voltage and current relationships are as follow :—

$$E = I_1(R_1 + j\omega L_1) + I_2 j\omega M \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

$$0 = I_1 j\omega M + I_2(R_2 + j\omega L_2) \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

$$I_2 = \frac{-j\omega M I_1}{R_2 + j\omega L_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

and substituting this in (25)

$$E = I_1 . (R_1 + j\omega L_1) - \frac{I_1 j^2\omega^2 M^2}{R_2 + j\omega L_2} \quad \ldots \quad \ldots (28a)$$

Noting that $j^2 = -1$ and rationalising the second factor in (28a)

$$E = I_1\left[R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2{}^2 + \omega^2 L_2{}^2}\cdot(R_2 - j\omega L_2)\right]$$

$$= I_1\left[R_1 + \left(\frac{\omega^2 M^2}{R_2{}^2 + \omega^2 L_2{}^2}\right)R_2 + j\omega\left(L_1 - \frac{\omega^2 M^2}{R_2{}^2 + \omega^2 L_2{}^2}L_2\right)\right]$$

$$E = I_1\,[R_1 + AR_2 + j\omega\,(L_1 - AL_2)] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(28b)}$$

where $A = \dfrac{\omega^2 M^2}{R_2{}^2 + \omega^2 L_2{}^2}$

If the section R_2L_2 were not short-circuited, expression 28b would be modified to

$$E = I'_1\,[R_1 + R_2 + j\omega(L_1 + L_2 + 2M)] \quad \ldots \quad \ldots \quad \ldots \text{(28c)}$$

In most r.f. and power coils X_L will be several times greater than R, and short-circuiting will therefore have greatest influence on the reactance, which is reduced, so increasing the current from the supply. The increased current inevitably leads to increased power loss, but the latter increase is much less for the r.f. coil than for the power transformer because A is very much less.

The value of the circulating current is calculated from (27), which may be rewritten in its modulus form as

$$|I_2| = \frac{\omega M}{\sqrt{R^2 + \omega^2 L_2{}^2}}\cdot|I_1|$$

$$= \sqrt{A}\,.\,|I_1| \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(29)}$$

When $R_2 \ll \omega L_2$

$$\sqrt{A} = \frac{M}{L_2} = \frac{M}{\sqrt{(L_1L_2)}}\sqrt{\left(\frac{L_1}{L_2}\right)} = k\sqrt{\left(\frac{L_1}{L_2}\right)}$$

Hence the circulating current is directly proportional to the coupling coefficient and the square root of the total to short-circuit turn inductance ratios.

To illustrate the above theory the following examples will be taken from r.f. and power practice.

R.F. Coil with Short-circuited Section

The r.f. coil has the following constants :—

$L_1 = 200\ \mu$H,
$L_2 = 50\ \mu$H,
k_2, (the coupling coefficient $\dfrac{M}{\sqrt{L_1L_2}}$) $= 0.3$,

$$Q = \frac{\omega L_1}{R_1} = \frac{\omega L_2}{R_2} = 100,$$

$$f = 1 \text{ Mc/s.}$$

$$\omega L_1 = \frac{6.28 \times 10^6 \times 200}{10^6} = 1256\ \Omega, \quad R_1 = 12.56\ \Omega$$

$$\omega L_2 = \frac{6.28 \times 10^6 \times 50}{10^6} = 314\ \Omega, \quad R_2 = 3.14\ \Omega$$

$$k = 0.3 = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{\sqrt{200 \times 50}}$$

$$M = 30\ \mu H$$

$$\omega M = \frac{6.28 \times 10^6 \times 30}{10^6} = 188.4\ \Omega$$

$$A = \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2} = \frac{3.55 \times 10^4}{(3.14)^2 + (314)^2} = 0.36$$

Impedance of coil with short-circuit $= \dfrac{E}{I_1} = (R_1 + AR_2) + j\omega\,(L_1 - AL_2)$

$$= (12.56 + 1.13) + j\omega\,(200 - 18) \times 10^{-6}$$

$$= 13.69 + j\omega\, 182 \times 10^{-6}$$

$$= 13.69 + j\, 1142 \text{ ohms}$$

$$|Z| = \left|\frac{E}{I_1}\right| = \sqrt{13.69^2 + 1142^2} \simeq 1142\ \Omega$$

When not short-circuited

Impedance $\dfrac{E}{I'_1} = R_1 + R_2 + j\omega\,(L_1 + L_2 + 2M)$

$$= 15.7 + j\omega\,(310 \times 10^{-6})$$

$$= 15.7 + j\, 1947$$

$$|Z| = \left|\frac{E}{I'_1}\right| \simeq 1947\ \Omega$$

Ratio $\dfrac{\textit{Current without short-circuit}}{\textit{Current with short-circuit}} = \dfrac{I_1}{I'_1} = \dfrac{1947}{1142} = 1.7$

Printed in England by
Electrical Press Ltd., Maidenhead.