

ANALOGUE INTEGRATED CIRCUITS FOR MODULATION AND MULTIPLICATION

This information sheet is intended as a guide to the properties of Motorola integrated circuits MC1594, MC1595 and MC1596. The examples of their use have been taken mainly from television equipment. However, they are also widely used in Audio, Instrumentation and other fields. The principles of operation of the circuits are given in an appendix.

The circuits are produced to both commercial and military specification. The commercial devices are labelled MC14xx and military devices are labelled MC15xx. Two of the types of multiplier are stocked by BBC valve stores, the MC1595 has a BBC number LIC795 and the MC1596 has the number LIC796.

The applications are presented in two parts:- part 1 - applications of four quadrant multipliers; part 2 - applications of a balanced modulator-demodulator.

MULTIPLIERS

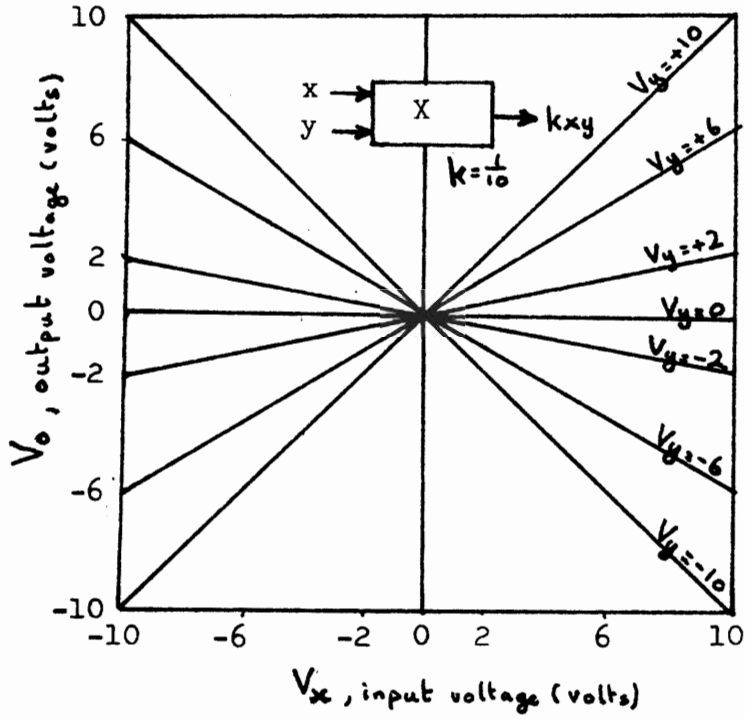
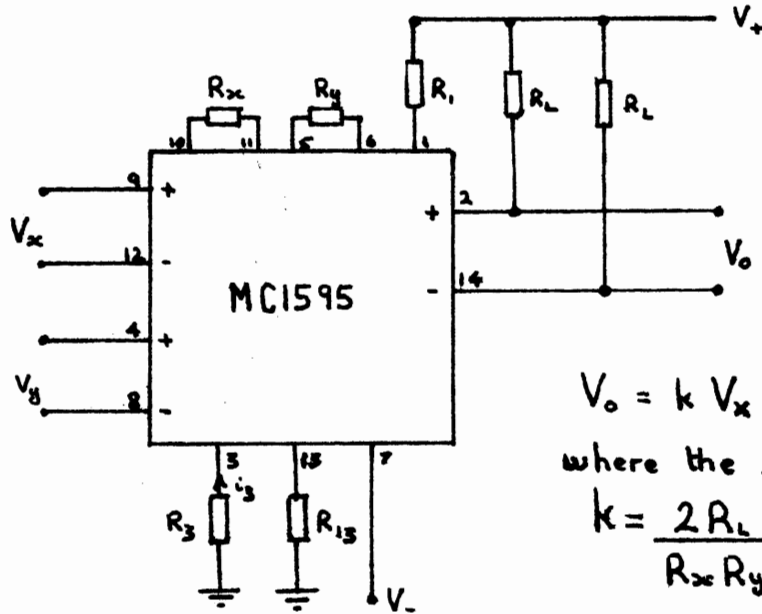


Figure 1

4 quadrant multiplier transfer function



$$V_o = k V_x V_y$$

where the scale factor is;

$$k = \frac{2R_L}{R_x R_y i_3}$$

Figure 2

Basic multiplier circuit

PART 1 : FOUR QUADRANT MULTIPLIERS

Motorola produce two four quadrant multiplier integrated circuits, the MC1594 and the MC1595. The input output transfer characteristic of these devices is shown in figure 1.

The MC1594 is a low bandwidth device, uniform gain as a function of frequency occur for frequencies (up to about 100kHz) and has a single ended voltage output. This I.C. is not normally used in video applications.

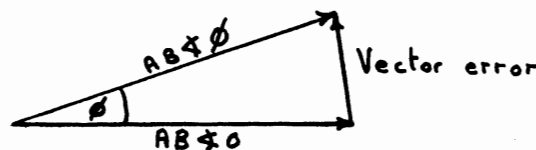
The MC1595 produces a differential current output which may be turned into a differential voltage by use of suitable load resistors. The basic circuit is shown in figure 2. To obtain maximum bandwidth from the I.C. the load resistance must be low. If it is below 50Ω the frequency response is flat to about 20MHz. Unfortunately the bandwidth, linearity and multiplicative scale factor, k, are interdependent. For instance, if the input signals are restricted to a maximum of ± 1 volt, and the bandwidth required is 20MHz, then the maximum scale factor is 0.09; if the bandwidth is restricted to 100kHz the maximum scale factor is 16.

Linearity is defined as :-

$$\text{Linearity} = \frac{\text{Max. Error}}{\text{Max. Output}} \times 100\%$$

The MC1595 set up to operate with $V_{\text{out}} = 0.1V_x V_y$, and with $V_x (\text{max}) = V_y (\text{max}) = \pm 10$ volts has a quoted x input linearity of ± 1% and a y input linearity of ± 2%.

Phase shifts may occur within the circuit for two reasons, either as a result of stray capacitance across the output load resistors or as a result of differences in the transadmittance of the x and y amplifiers. The MC1595 is quoted to have a 3° relative phase shift between the x and y inputs at a frequency of 750kHz. The phase shift is sometimes quoted as a vector error, where the vector error is that vector which joins the expected output vector to the actual output vector. If the inputs are A and B we expect an output $AB \angle 0$ and we find the output is $AB \angle \phi^*$. The vector error is the vector joining these two as is shown below.



(* Note $AB \angle \phi$ means phasor AB at an angle ϕ)

MULTIPLIERS

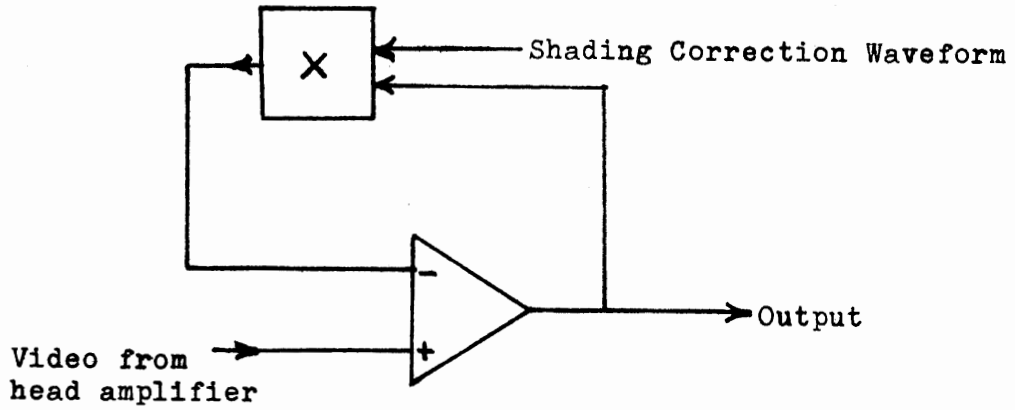


Figure 3 Schematic of white shading correction, Link 110 camera.

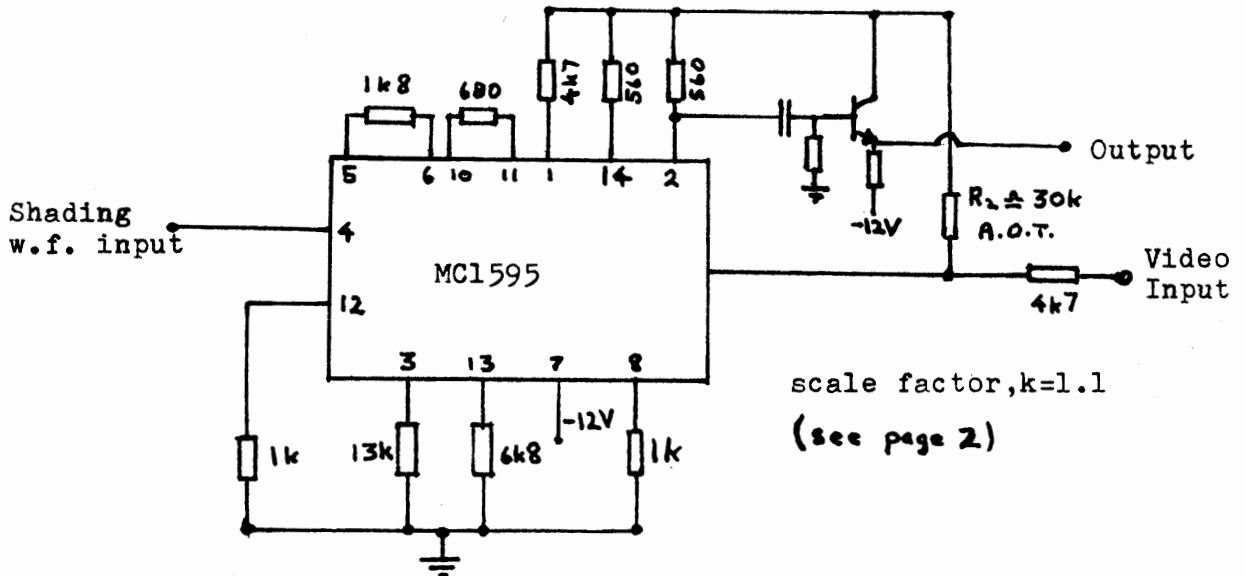


Figure 4 Detail of multiplier connection, Link 110 camera

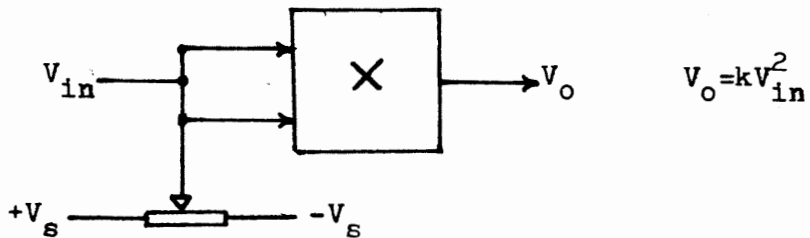


Figure 5 Basic squaring circuit

APPLICATIONS

Shading Corrector

A common use of multipliers in television equipment is the correction of white shading in a camera. Figure 3 shows how this is done in the Link 110 camera. Detail of the multiplier circuitry is shown in figure 4. This correction is applied in the red, green and blue video processing amplifiers. Enclosing the multiplier within a feedback loop ensures that non-linearities or phase shifts caused by the multiplier have minimum effect on the output video. The EMI 2005 camera uses a shading corrector very similar to that of the Link, the major difference being that correction is applied to the red and blue channels only and is adjusted to obtain the same shading as may be present in the green signal.

The Phillips LDK5 was designed before the MC1595 became available but uses discrete components and transistor arrays connected as a multiplier.

Parabola Generator

If the same voltage is applied to both inputs of the multiplier then the will be proportional to the square of the input. If the input voltage consists of a signal input voltage V_i and a d.c. voltage V_{dc} ,

i.e.
$$V_{in} = V_i + V_{dc}$$

then
$$V_{out} = (V_i + V_{dc})^2 = V_i^2 + 2V_{dc} V_i + V_{dc}^2$$

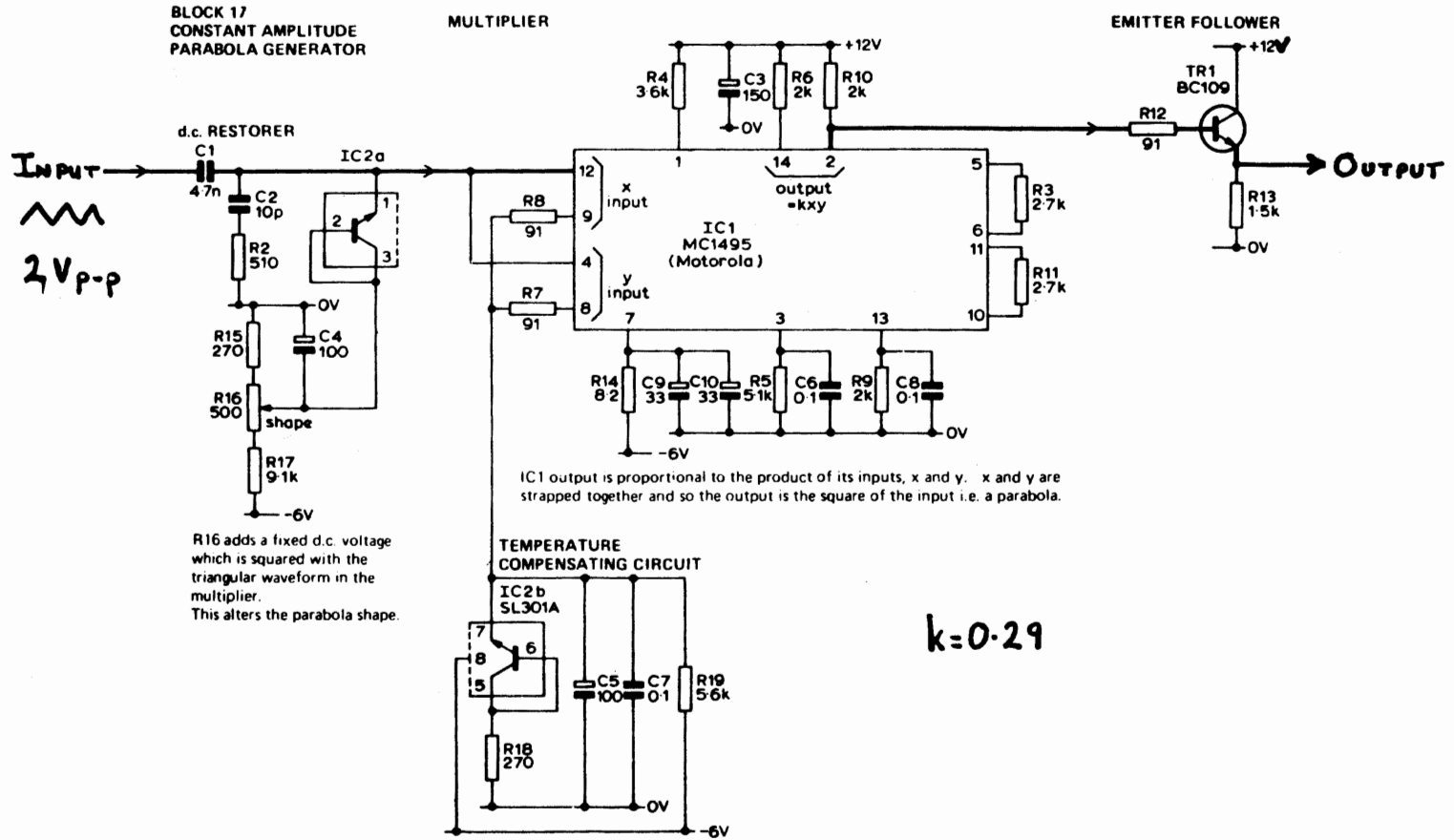
Therefore it is normal to provide some method of adjusting the d.c. conditions at the input.

The basic squaring circuit is shown in figure 5.

Note a most useful result that if the signal input is a triangular wave, and the d.c. condition is correct, then the output will be a parabola.

Figure 6

Parabola generator used in BBC vision mixer



An MC1495 is used for this purpose in the BBC studio vision mixer EP5/512. The circuit used to generate the line parabola is shown in figure 6. The field parabola generator differs from the line parabola only in minor design points in the d.c. restorer.

Frequency Multiplier

From trigonometry we know that :

$$\cos^2 \phi = 1 + \cos 2\phi$$

Thus if we apply a sine wave to both inputs then frequency doubling will occur. If the input is $V_{in} = E \cos (wt)$ then the output will be

$$V_o = kE^2 + kE^2 \cos (2wt)$$

which is a sine wave of twice the input frequency superimposed on a d.c. potential.

Higher powers may be obtained by cascading multipliers. The problem with this is any thermal drift present in the first stage is magnified in subsequent d.c. coupled stages. As an example we could consider the problem of driving an analogue timebase corrector. The capacitance of the reverse biased diodes used in 'Amtec' and 'Colortec' varies as (Applied Voltage)⁴, thus if we were to cascade two multipliers we could obtain a suitable drive voltage for the diodes from the input timing error voltage.

Modulator

If we apply two inputs

$$V_x = E_c \cos (w_c t)$$

and $V_y = E_m \cos (w_m t)$

to the multiplier of figure 2, then the output voltage is given by

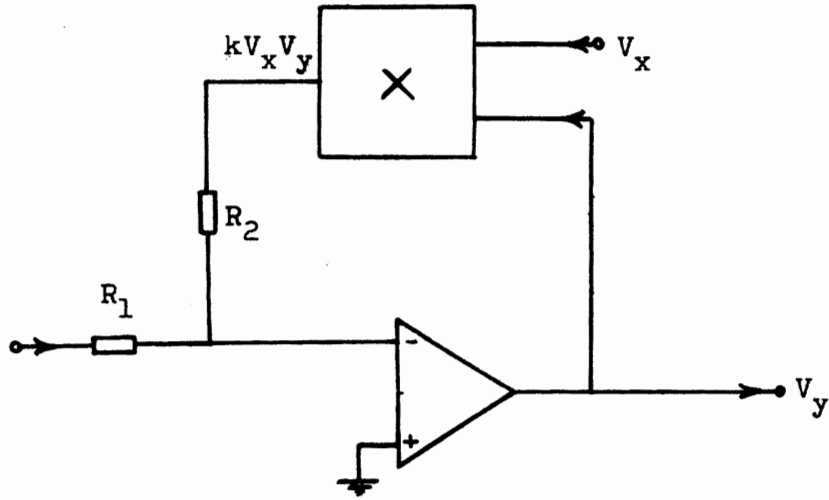
$$V_o = k \left\{ E_c \cos (w_c t) \right\} \left\{ E_m \cos (w_m t) \right\}$$

This may be re-written as

$$V_o = \left\{ \frac{kE_c E_m}{2} \right\} \left\{ \cos (w_c + w_m)t + \cos (w_c - w_m)t \right\}$$

which is the equation for balanced modulation.

MULTIPLIERS



$$V_y = -\frac{R_2}{R_1} \cdot \frac{1}{k} \cdot \frac{V_z}{V_x}$$

If $R_1 = R_2$

$$V_y = \frac{-V_z}{kV_x}$$

If $R_2 = kR_1$

$$V_y = \frac{-V_z}{V_x}$$

Figure 7 The basic divide circuit

We may also use the multiplier to recover a single sideband modulated signal provided that we know the carrier frequency.

Let
$$e_{ssb} = A \cos (\omega_c + \omega_m)t,$$

and let the reinserted carrier be

$$e_c = E_c \cos (\omega_c t)$$

then the product is given by

$$e_c e_{ssb} = AE_c \cos (\omega_c t) \cos (\omega_c + \omega_m)t$$

therefore
$$V_o = k e_c e_{ssb} = \frac{kAE_c}{2} (\cos(2\omega_c + \omega_m)t + \cos(\omega_m)t)$$

if the output signal is now passed through a low pass filter we are left with the wanted modulating signal.

Suppose we add a d.c. offset to the modulation input of a circuit set up for balanced modulation then the output V_o is given by:-

$$V_o = kE_c E_{dc} \cos (\omega_c t) + kE_c E_m (\cos(\omega_c + \omega_m)t + \cos (\omega_c - \omega_m)t)$$

The first term, $kE_c E_{dc} \cos (\omega_c t)$, represents residual carrier whose magnitude may be varied by the voltage V_{dc} . Thus the added d.c. voltage is a peak modulation depth control for an amplitude modulated signal.

Gain Control

If the inputs are fed with a signal voltage, V_s , and a control voltage, V_c , then we get $V_o = k V_s V_c$.

This tells us that the output can be linearly controlled in amplitude by V_c .

Uses for the technique include remote control of gain, automatic gain control, use in an audio compressor and the shading corrector mentioned earlier.

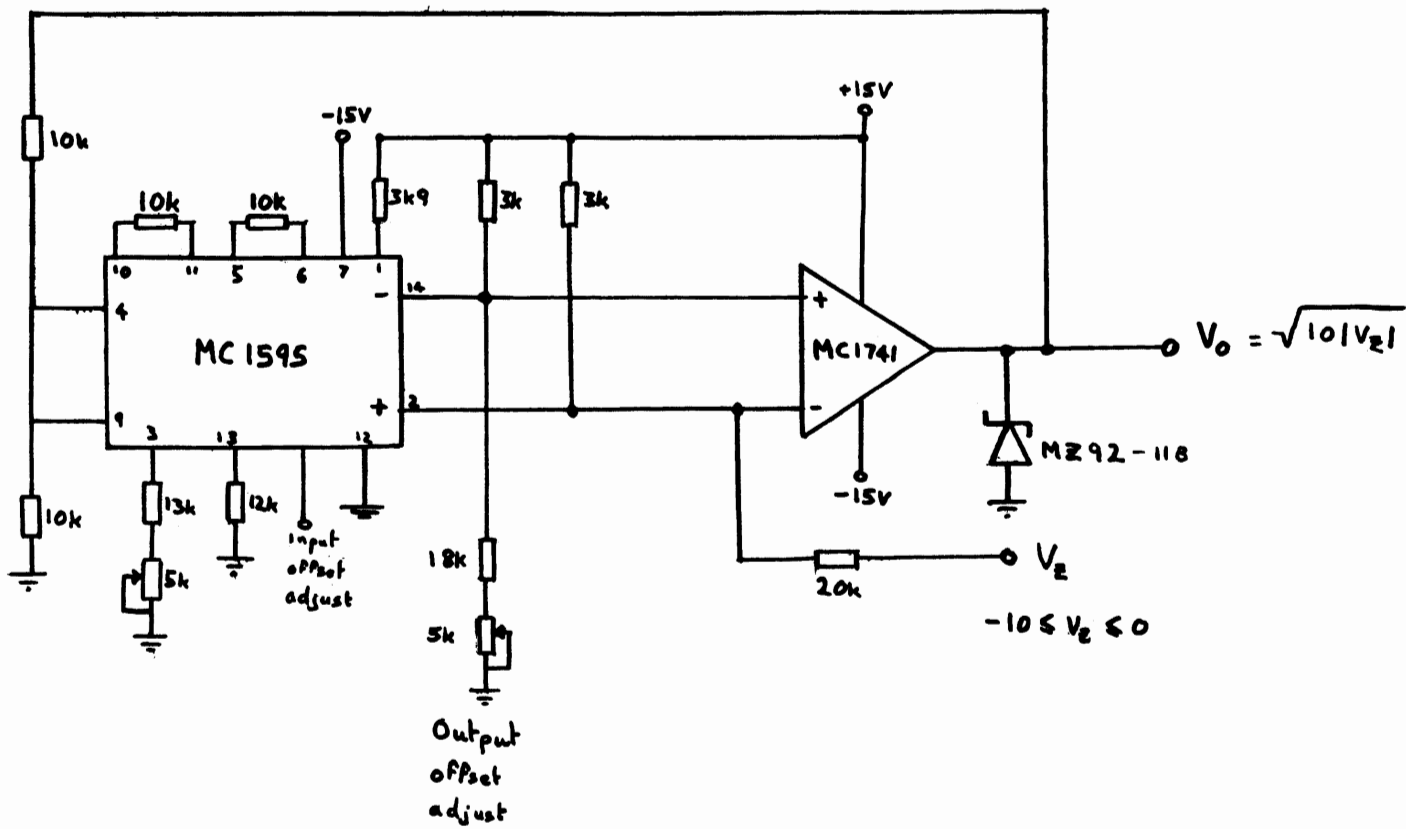


Figure 8

An example of a square root circuit

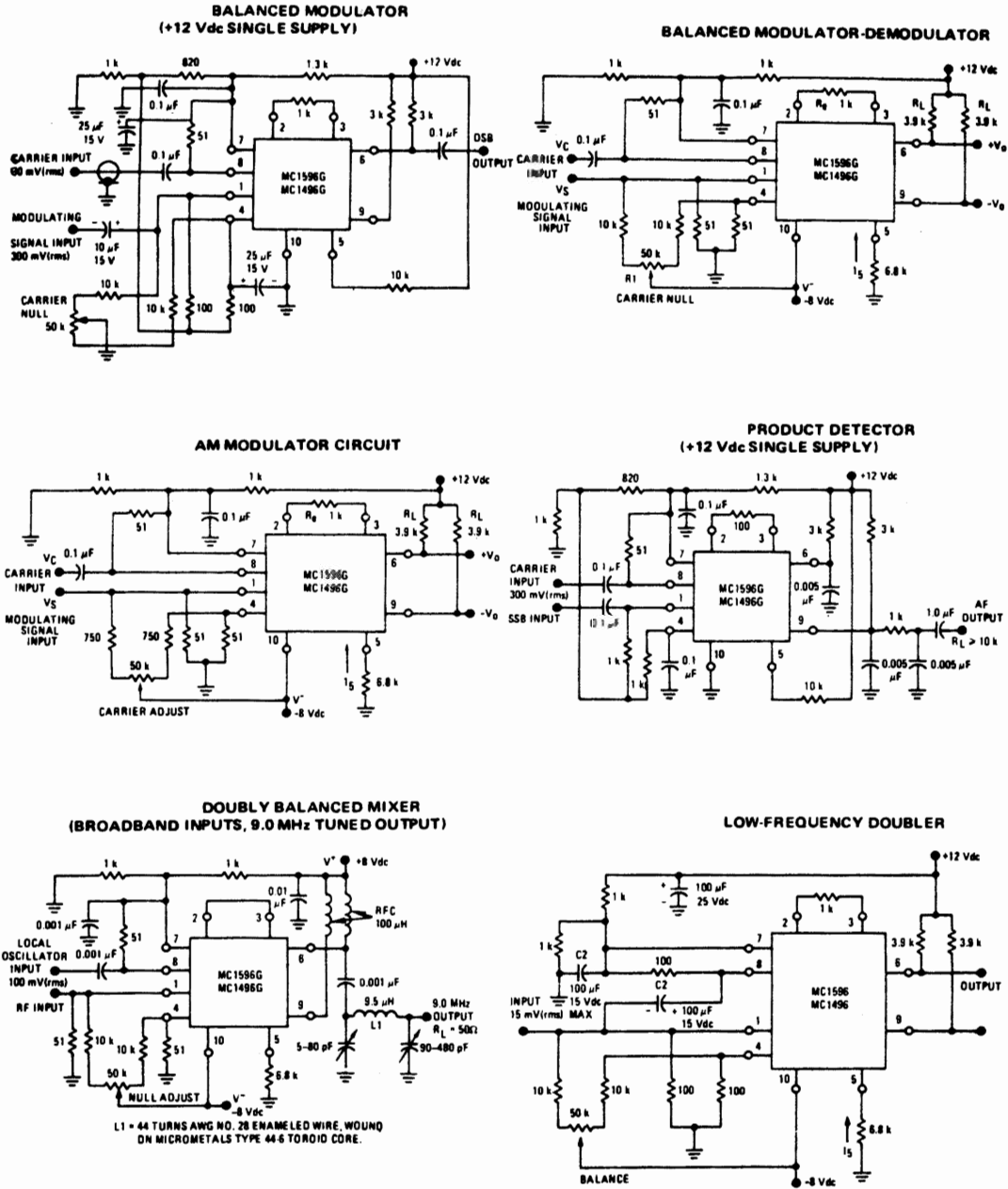
Reciprocals and Square Roots

If the multiplier is enclosed within the negative feedback loop of an operational amplifier as shown in figure 7 then voltage division results. If, in the circuit of figure 7, the voltage V_z is fixed, then the output, V_y , is proportional to the reciprocal of the voltage V_x .

If both inputs of the multiplier are connected to the output of the operational amplifier then the output, V_y , will be proportional to the square root of V_z . Care must be taken to ensure that V_z is of the correct polarity to avoid the possibility of output latch up. A possible square root circuit is shown in figure 8.

MULTIPLIERS

TYPICAL APPLICATIONS



Pin number references pertain to this device when packaged in a metal can.

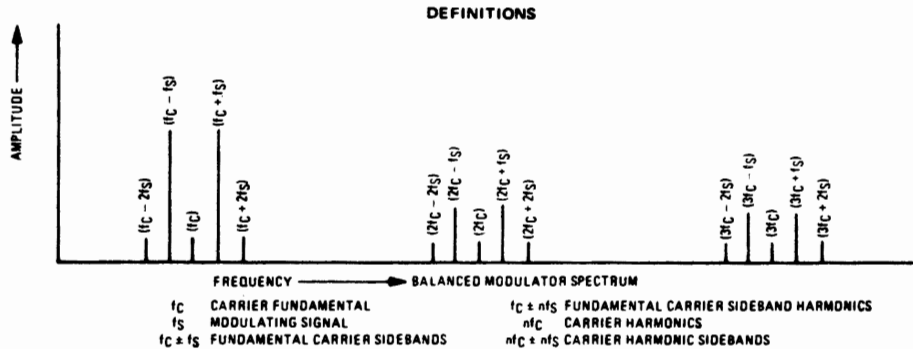


Figure 9

Various circuits using the MC1596

The MC1596 is a wideband multiplier-modulator. It is designed to operate with a switching carrier input of up to 300 MHz and a linear signal input. However, with careful circuit design, it can be used in a similar fashion to the MC1595 when the wider bandwidth is essential. (see appendix).

The bandwidth of the MC1596, as in the MC1595, depends on the load resistance. For instance, if the MC1596 is operated with a single ended voltage gain of +18dB ($R_L = 3.9 \text{ kohms}$), then the frequency response measured at the output to a voltage at the signal port is flat to 1MHz and is 2dB down at 5MHz. If the signal port transadmittance, defined as

$$y_{21} = \frac{I_{\text{out}}}{V_{\text{in}}} \quad \left| \quad V_{\text{out}} = 0 \right.$$

is measured it is found that the transadmittance is constant to 10MHz falling to half the value at 250MHz. Measurement at $V_{\text{out}} = 0$ implies that the load is a perfect current sink. It is therefore common when a large bandwidth is required to provide a common base transistor stage as the load.

The carrier port has a frequency response extending to 300MHz assuming that the load is suitable.

A schematic circuit diagram of the MC1596 is shown in figure A2 of the appendix. Many of the simple applications of the MC1596 are the same as the modulator applications of the MC1595. Figure 9 shows several possible uses and also shows the spectrum of a balanced modulator.

APPLICATIONS

The MC1596 is much used in current BBC designed equipment and appears in many configurations. Therefore the examples given do not represent a comprehensive survey but may be taken as a guide to the many uses of the I.C.

MULTIPLIERS

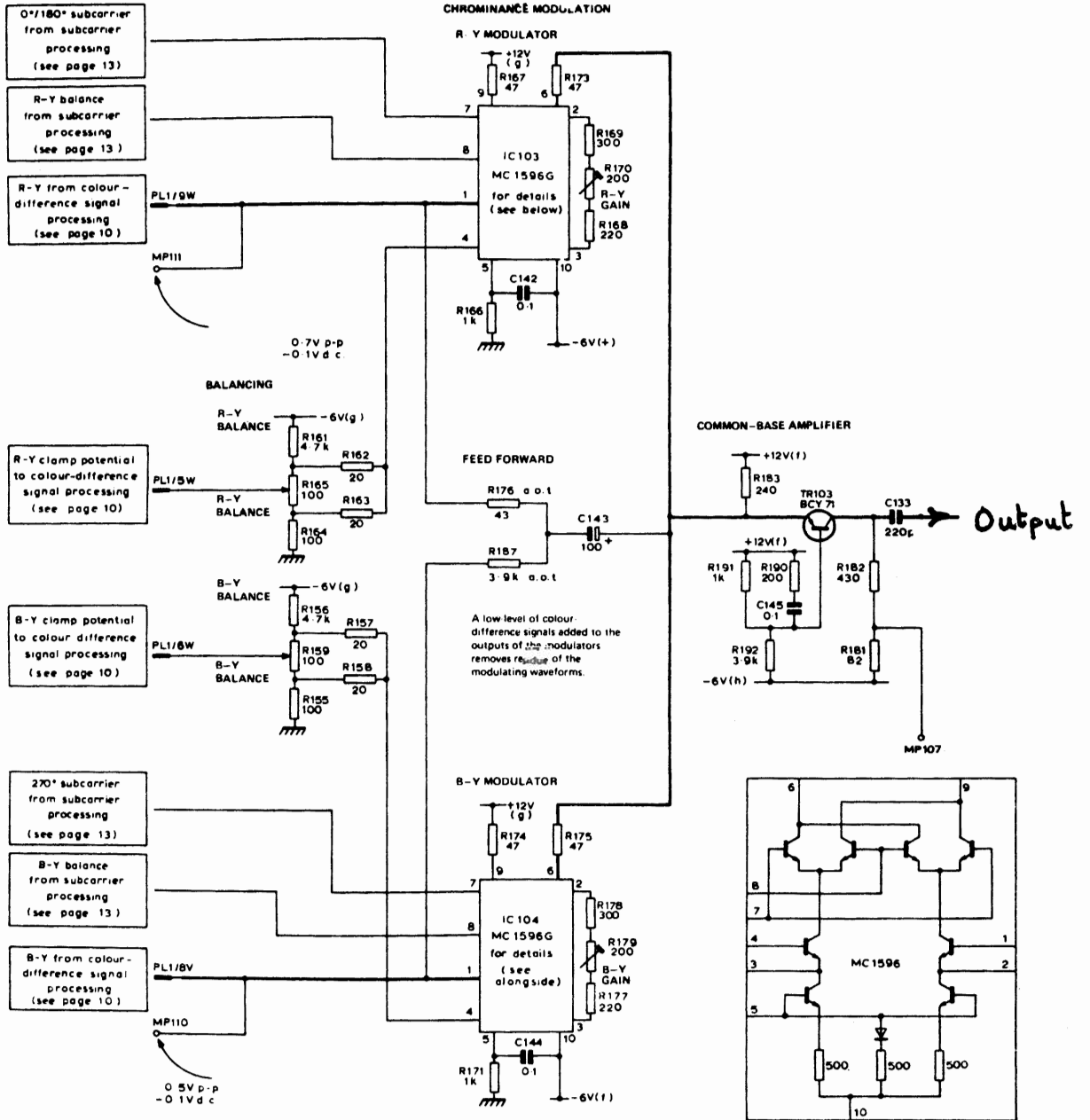


Figure 10 Detail of chrominance modulator, BBC PAL coder

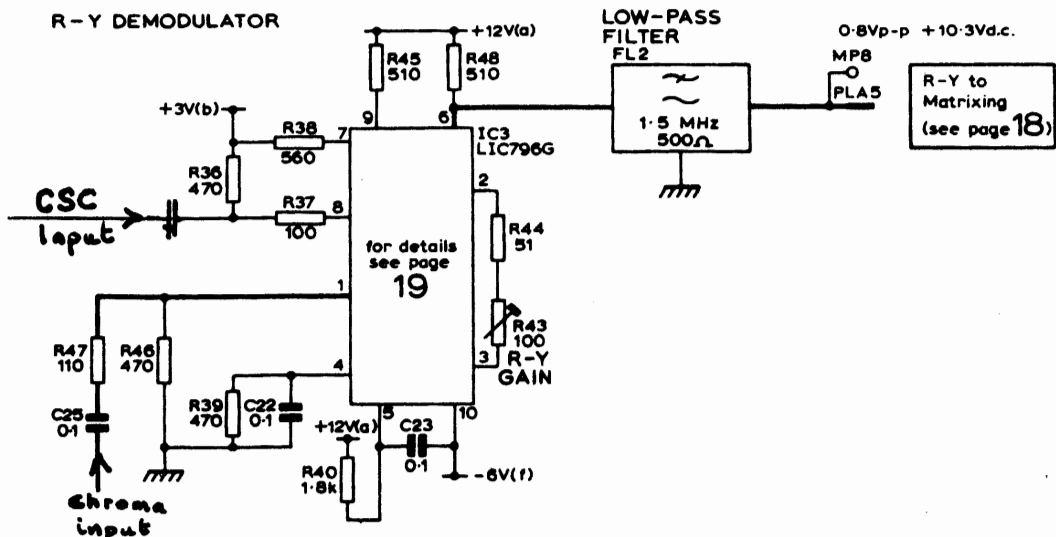


Figure 11 Detail of demodulator, BBC PAL decoder

EXAMPLES OF USE AS A MODULATOR

The first example is taken from the BBC PAL coder CD2/503. Figure 10 shows the (R-Y) and (B-Y) modulators. The MC1596 is used in its saturated mode, therefore the output voltage depends only upon the input signal current and the load resistance. As is shown in the appendix the signal current produced can be controlled by the emitter degeneration resistor connected between pins 2 and 3. The d.c. conditions at the signal input are adjusted by changing the clamp potential of the colour difference signals. The (R-Y) and (B-Y) balance consists of a d.c. voltage set to be equal to the mean d.c. level of the colour subcarrier. To improve stability this voltage is produced by drive circuits with the same temperature dependence as the subcarrier drive circuit.

The signal formed on load resistors R182 and R183 is passed through a filter. This removes 2nd and 3rd harmonics of the carrier by tuned notches and higher frequencies by a low pass section.

The MC1596 is used as a demodulator in the BBC PAL monitoring decoder CD3/505. The circuit of the (R-Y) demodulator is shown in figure 11. The (B-Y) demodulator is identical.

It is used as a modulator and demodulator in the black edge caption generator of the BBC vision mixer EP5/512. The modulator used here is to provide a signal suitable for transmission along an ultrasonic delay line. The carrier frequency used is 6.4MHz.

MULTIPLIERS

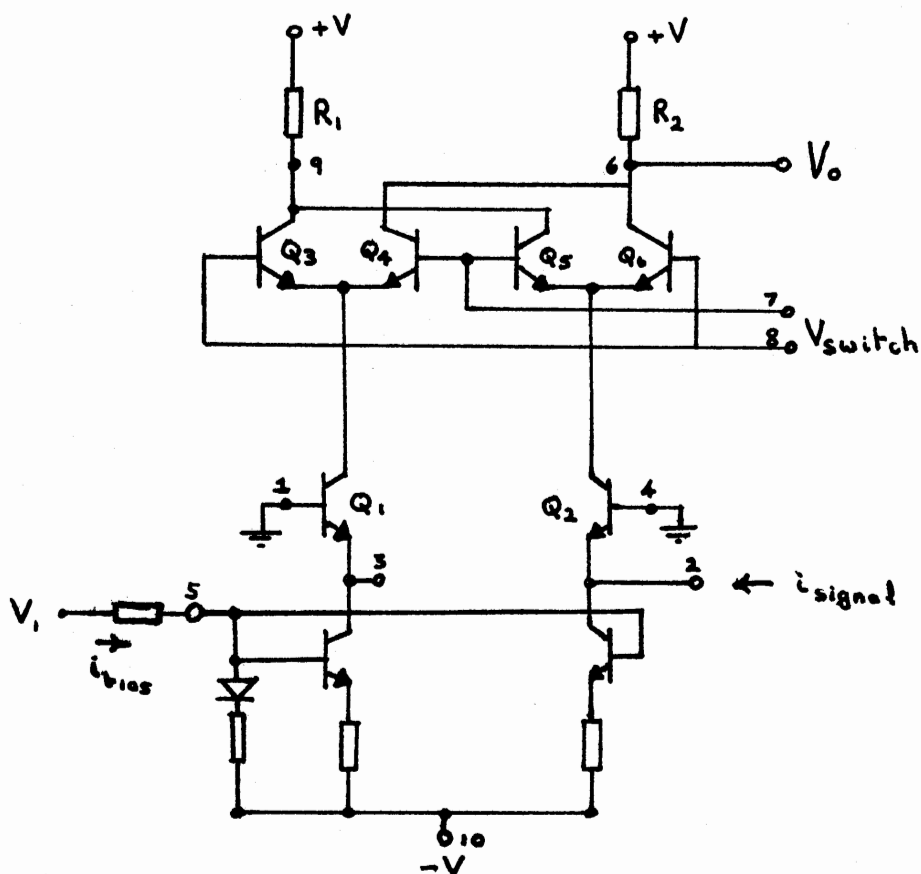


Figure 12 Schematic of the MCL596 used as a high speed switch

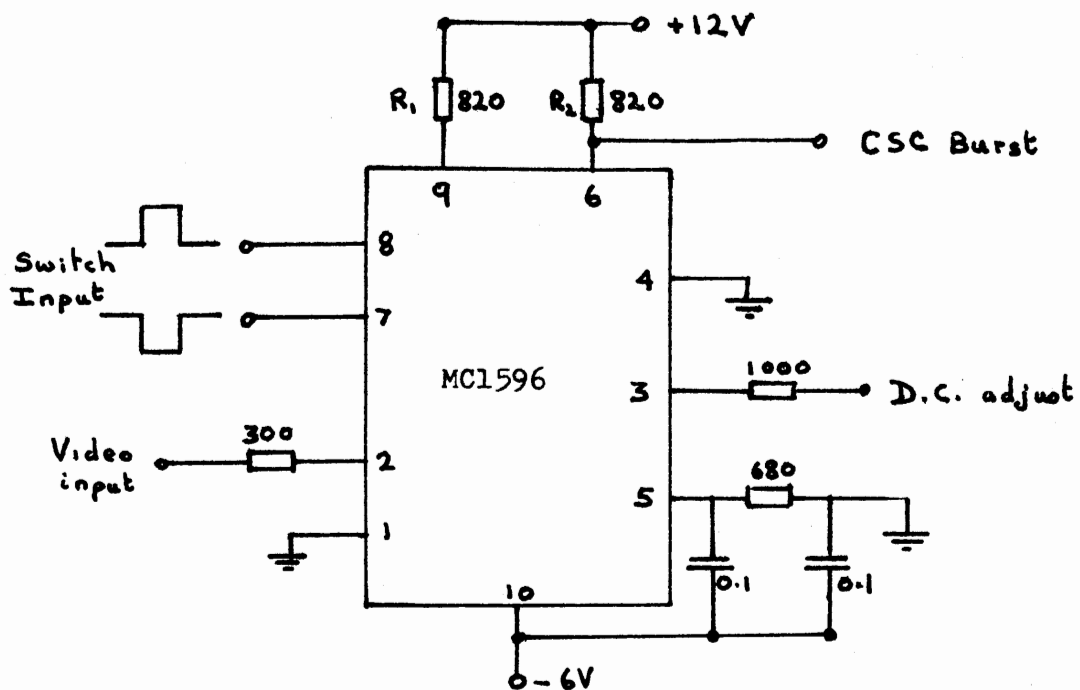


Figure 13 Burst separator used in the BBC vision mixer

Video Switch

The MC1596 can be used as a high speed switch. Consider the circuit shown in figure 12. In this circuit we inject a signal current direct into the common base amplifier Q2, but any other method of generating a signal current from the signal voltage may be used. The signal current produced must flow in the collector circuit of Q2. The transistors Q5 and Q6 act as a single pole double throw switch thus the collector current of Q2 is switched between R1 and R2. The transistors Q1, Q3 and Q4 set the d.c. conditions of R1 and R2 when they are not switched to the signal current.

An example of this use as a switch is the burst separator used in the BBC vision mixer. The connections to the MC1596 are shown in figure 13. Pulses of opposite polarity and 3.5 sec. width, derived from the trailing edge of mixed sync, are applied to the carrier inputs of the MC1596. The video is injected as a current into pin 2. During the time at which the burst is expected the video current is switched from R_1 to R_2 . A d.c. current is injected at pin 3 and adjusts the voltage across R_2 such that the burst is placed symmetrically about the output d.c. level.

If, in the circuit of figure 13, the d.c. current is replaced by a second video current switching between the two inputs will occur. To adjust the output d.c. level of this circuit the current injected into the bias input (pin 5) of the MC1596 is adjusted, thus changing the standing current in the output load. Several examples of this type of switch are found in the BBC vision mixer. For more details see vision mixer handbook section

- 3.9 - Overlay split screen switch
- 3.11 - A Bank/B Bank split screen switch
- 4.5 - Insert correct sync and burst into output video, add to this video cue dot, caption letters and I.T.S.

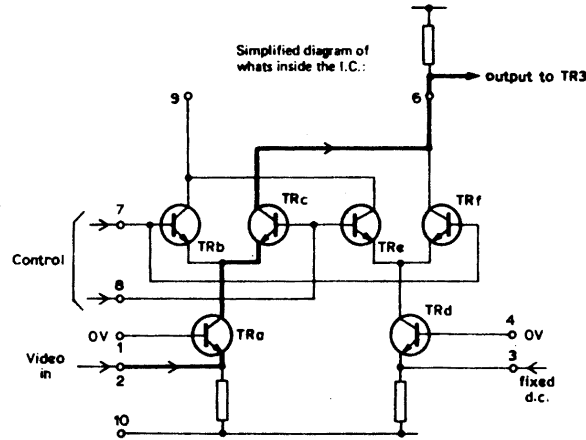
If the emitters of the signal input transistors (Q1 and Q2 of figure 12) are coupled by a resistor and a signal voltage applied between the bases of these transistors, opposite senses of signal current are produced in their collector circuits (see appendix). The load resistor in which each signal current flows may be selected by the switching transistors (Q3, Q4, Q5 and Q6 of figure 12). Since the currents are of opposite sense then the output may be inverted by switching. For more details see vision mixer handbook section 8 - 15.

BLOCK 17 GATED FADE

Figure 14

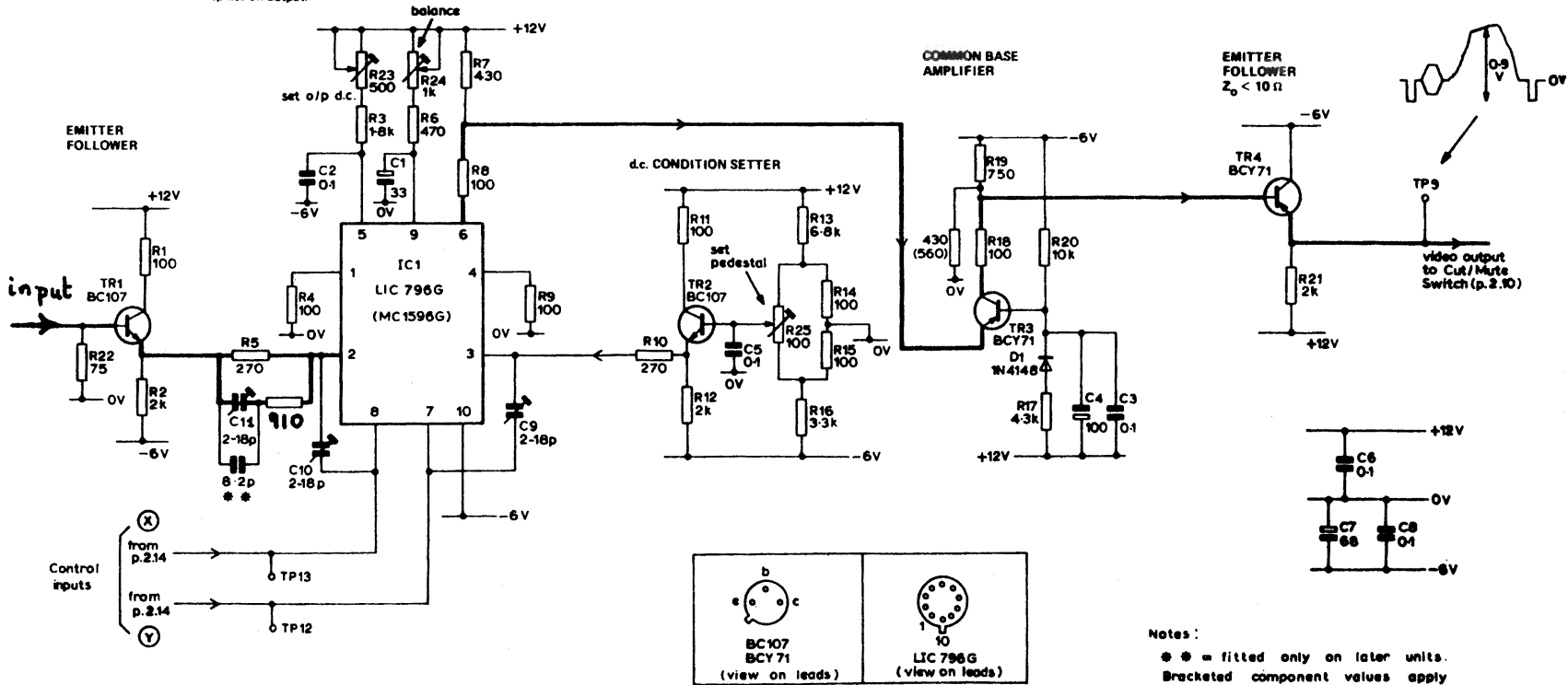
The MC1596 used as a gated fader in the BBC vision mixer

MULTIPLIERS



How it works:
 TRa acts as a Common Base amplifier and it drives the emitters of a long-tail pair, TRb, TRc.
 D.C. control signals at pins 7 and 8 control the degree of conduction of TRb and TRc.
 TRc thus acts as a series variable resistor in the video chain, its resistance depending upon the d.c. voltages at pins 7 and 8 (e.g. with pin 8 more positive than pin 7 TRc conducts hard and acts as a low-value resistor).
 TRc drives a low input-impedance Common Base stage, TRd and so the resistance of TRc has a large effect on the signal current into and hence the signal voltage out of TRd.
 TRd, e and f play no part in the fading process and the inputs to the base and emitter of TRd serve only to keep the d.c. conditions correct.

FADING ELEMENT
 C11 adjusts h.f. response
 C9 and C10 are set for minimum spikes on output.



Video Fader

If the carrier input port is not driven into saturation then the signal input current is multiplied by a function depending on the voltage at the carrier port (see appendix). In this linear mode the output current is given by $I_o = I_{in} f(V_c)$, where $f(V_c)$ is a non linear function of V_c and may take values between 0 and 1. Thus if we control the voltage V_c such that $f(V_c)$ goes from 1 to 0 in a smooth fashion the output will reduce from maximum to zero. This fade action is used in vision mixing.

The circuit of a channel fader is shown in figure 14. It will be remembered that the transconductance of the signal port is constant up to frequencies above the video band if the output is into a low load resistance. In the channel fader this low resistance is provided by the common base amplifier TR3. The capacitor C11 provides a small of h.f. boost (0.3dB - 3dB at 5.5MHz) to compensate for stray capacitance. The output current is converted to an output voltage by the collector load of TR3. The linear control voltage from the fader has its shape distorted to provide a suitable fader law. End stop detectors on the faders ensure that the switching voltage on pins 7 and 8 is hard on in the appropriate sense when the fader is at the top or bottom of its travel. The switching voltage can also be arranged to turn the output to full during blanking thus preserving sync and burst at full amplitude. This is known as 'Gated Fading'.

If a split screen or wipe is required this can be obtained by performing a rapid cross fade from one input to another. If this cross fade is done relatively slowly a soft edge will result, the slower the crossfade the softer the resulting edge. This use as a gated fader in both fast (e.g. burst separator) and the fast/slow (e.g. channel fader) modes occurs at many places within the vision mixer. For more examples see the EP5/512 vision mixer handbook sections 2 - 9; 2 - 22; 2 - 32; 3 - 6; 4 - 9; 5 - 7; 7 - 19; 8 - 17.

MULTIPLIERS

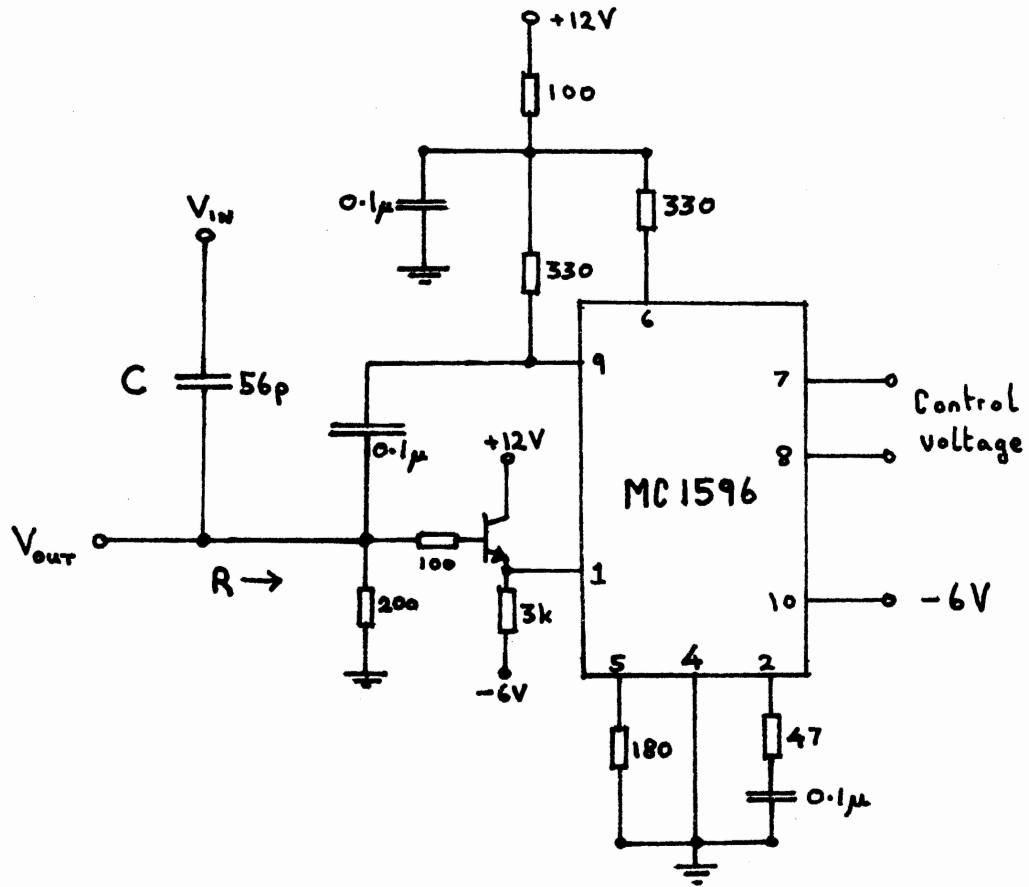


Figure 15 Variable input resistance amplifier

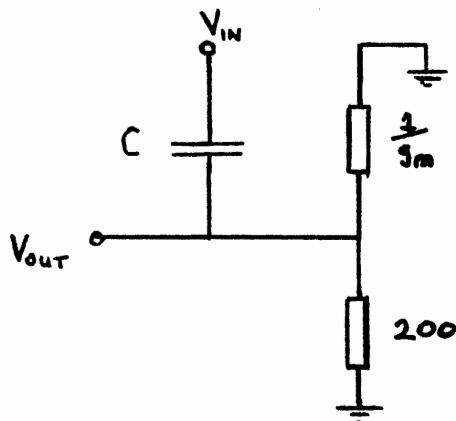


Figure 16 Essential features of the equivalent circuit

Small Angle Phase Shifter

If an amplifier has 100% voltage feedback applied the input resistance depends on the gain of the amplifier. The MC1596 can be used in this configuration and since the gain can be varied by a control voltage the input resistance can be varied by this voltage.

This circuit configuration is used in the auto phase corrector of the BBC vision mixer. If the input resistance is made the R of a CR circuit then the fraction of the input voltage (and hence the phase) appearing across the resistor may be varied by the control voltage. The control voltage may either be adjusted manually or a phase discriminator may be used to compare reference colour subcarrier with incoming burst and the pulse produced used to provide the control voltage for the amplifier. The circuit used for variable input impedance is shown in figure 15 and its equivalent circuit in figure 16.

Phase Discriminator

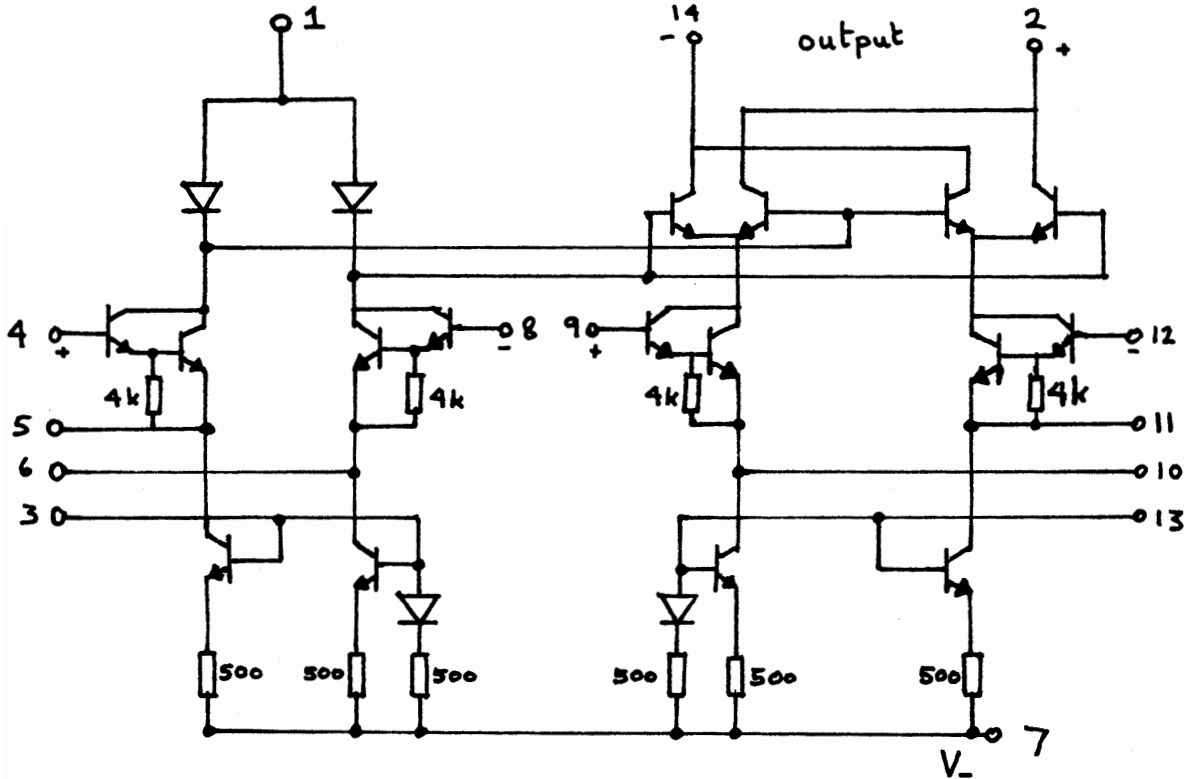
The MC1596 used as a synchronous demodulator can also act as a Phase Discriminator. If the two inputs are of the same frequency but differing phase then the output amplitude depends on the phase difference.

Consider two inputs $e_1 = E_1 \cos (wt + \phi)$, $e_2 = E_2 \cos (wt)$

$$\begin{aligned}
 \text{then } V_o &= k(E_1 \cos (wt + \phi)) (E_2 \cos (wt)) \\
 &= kE_1 E_2 (\cos(2wt + \phi) + \cos \phi) \\
 &= kE_1 E_2 \cos \phi + \frac{kE_1 E_2}{2} \cos (2wt + \phi)
 \end{aligned}$$

The first term of this expression is a d.c. potential whose magnitude and sign depend on the cosine of the phase difference . The second term is a sine wave of twice the input frequency. Using a suitable low pass filter will leave only the d.c. component. There are many applications of this technique. For a particular example refer to Burst Phase Comparison.

MULTIPLIERS



Pins 4 and 8 form the Y input
Pins 9 and 12 form the X input

Figure A1 Schematic of the Motorola MC1595L

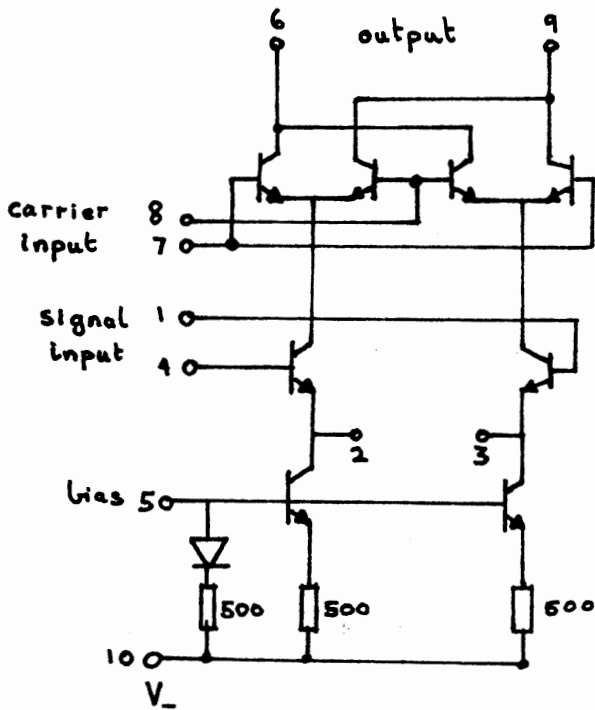


Figure A2 Schematic of Motorola MC1596G

APPENDIX

In this appendix the basic operation of the multiplier and modulator is considered. Figure A1 shows the schematic circuit of the MC1595. The MC1594 is nearly identical to the MC1595 in the operation of its multiplier circuitry. Figure A2 shows the schematic circuit of the balanced modulator MC1596. It should be noted that the right hand side of figure A1 is in principle the same as the circuit of the MC1596 with a carrier input signal derived from a second differential amplifier. Therefore we shall first consider the operation of the MC1596 and then consider the modifications needed to make an MC1595.

Analysis of the MC1596

The circuit diagram for analysis is shown in figure A3. Transistors Q_1 and Q_2 act as emitter followers, therefore the current i_x is given by

$$i_x = \frac{V_x}{R_x} \quad (\text{Assuming } R_x \gg r_e)$$

Hence the collector currents will be as shown in figure A3.

Suppose initially no differential voltage, V_y , is applied then by symmetry

$$i_2 = i_3 = \frac{1}{2}(i_1 + i_x) = \frac{1}{2}i_1 (1 + x) = i'$$

and
$$i_4 = i_5 = \frac{1}{2}(i_1 - i_x) = \frac{1}{2}i_1 (1 - x) = i''$$

Now let a non zero voltage, V_y , be applied. The currents in the transistors Q_3, Q_4, Q_5, Q_6 will vary according to the emitter dynamic resistance. Assume that this is such that the current in Q_4 increases by a fraction y , i.e. $i_3 = i'(1 + y)$. Summing currents at the emitters of Q_3 and Q_4 requires that $i_2 = i'(1 - y)$. Similarly $i_4 = i''(1 + y)$, $i_5 = i''(1 - y)$. The output from the MC1596 is a differential current $I_d = i_a - i_b = i_2 + i_4 - i_3 - i_5$.

$$\begin{aligned} I_d &= \frac{1}{2}i_1 (1+x)(1-y) + \frac{1}{2}i_1 (1-x)(1+y) - \frac{1}{2}i_1 (1+x)(1+y) - \frac{1}{2}i_1 (1-x)(1-y) \\ &= -2i_1 xy \end{aligned}$$

Remembering that $x = \frac{i_x}{i_1} = \frac{V_x}{i_1 R_x}$

$$I_d = \frac{2V_x \cdot y}{R_x} = \frac{2V_x \cdot f(V_y)}{R_x} \quad \text{where } f(V_y) \text{ is the function relating } y \text{ and } V_y.$$

MULTIPLIERS

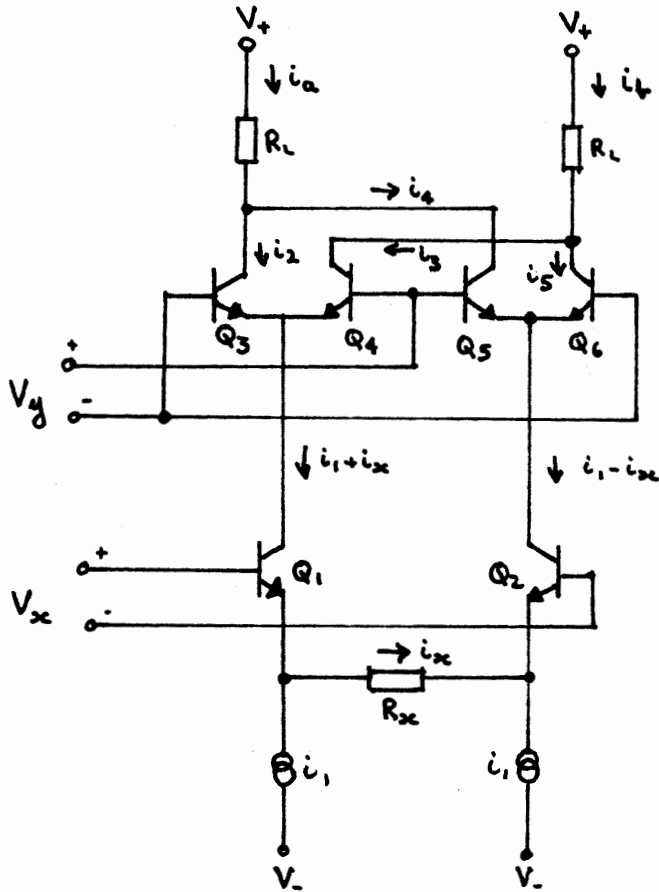
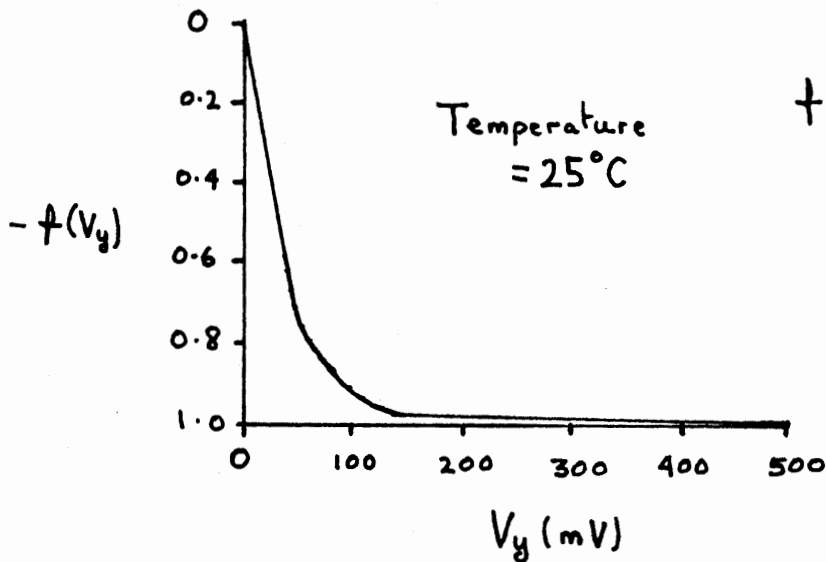


Figure A3 Analysis model of MC1596



$$f(V_y) = \frac{e^{-m} - e^m}{(1+e^m)(1+e^{-m})}$$

$$m = \frac{q}{kT} V_y$$

Figure A4 Showing voltage dependence of $f(V_y)$

Thus it can be seen that the output current is proportional to the product of V_x and some function of the voltage V_y . To determine the function $f(V_y)$ requires a more detailed analysis. The required analysis may be found in Motorola application note AN531. The main result found there is that

$$(1) \quad I_d = \frac{2V_x}{R_x} \frac{e^{-m} - e^m}{(1+e^m)(1+e^{-m})} \quad \text{where } m = \frac{q}{kT} \cdot V_y$$

The function $f(V_y)$ is plotted in figure A4. We expect that the differential output current may be written as

$$I_d = \frac{2V_x}{R_x} \cdot V_y \cdot \beta \quad (\text{where } \beta \text{ is a constant})$$

for that part of figure A4 which is linear. Thus we may expect linear operation of the multiplier for $V_y \leq 50\text{mV}$. For any input $V_y \geq 200\text{mV}$ the quad amplifier (Q_3, Q_4, Q_5, Q_6) becomes saturated and the circuit then becomes equivalent to that shown in figure A5. The output will then consist of a signal linearly proportional to the modulating input multiplied by a square wave of frequency f_c .

The square wave may be written $s(t) = 2 \sum_{n=1}^{\infty} A_n \cos(n\omega_c t)$

Therefore the output may be written

$$V_o = kE_m \sum_{n=1}^{\infty} A_n (\cos(n\omega_c + \omega_m)t + \cos(n\omega_c - \omega_m)t)$$

$$\text{where } A_n = \left[\frac{\sin(\frac{1}{2}n\pi)}{\frac{1}{2}n\pi} \right] \quad \text{N.B. for } n \text{ even } A_n = 0$$

The output will contain frequencies $\omega_c \pm \omega_m, 3\omega_c \pm \omega_m, 5\omega_c \pm \omega_m$ etc

If we chose a suitable filter then we may select from the output spectrum any of the desired frequencies.

Analysis of the MC1595

Figure A6 shows the y input amplifier of the MC1595. This amplifier operates in the same way as the x input amplifier.

Hence $i_y = \frac{V_y}{R_y}$, and the diode currents are as shown.

$$\text{For a diode } I = I_o e^{\left[\frac{eV}{kT} \right]}$$

$$\text{taking logs } \ln \left[\frac{I}{I_o} \right] = \left[\frac{e}{kT} \right] V \quad \text{also } \ln \left[\frac{I'}{I_o} \right] = \left[\frac{e}{kT} \right] V'$$

MULTIPLIERS

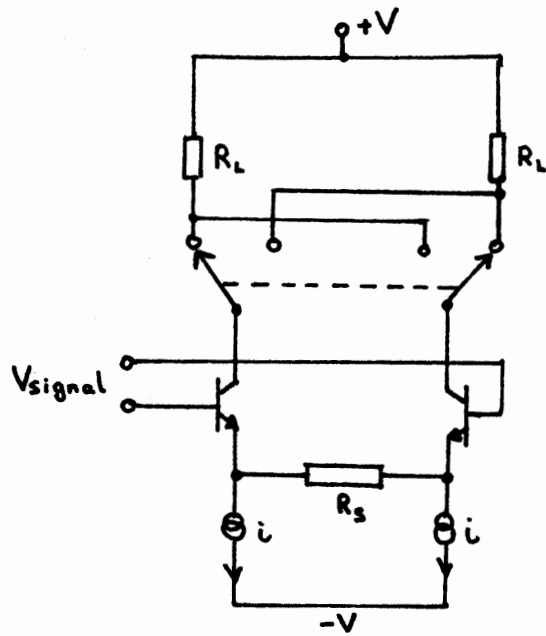


Figure A5 Equivalent circuit MCl596, saturated mode

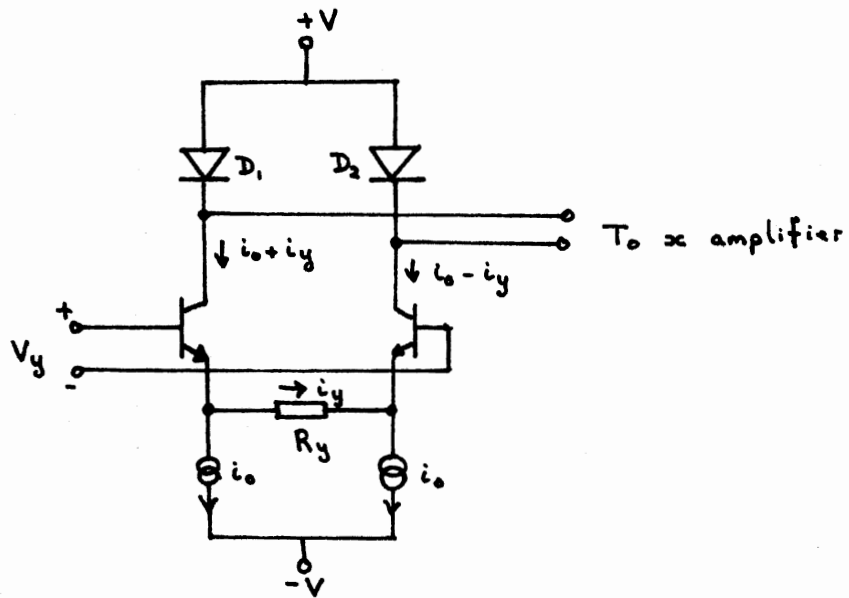


Figure A6 Y input amplifier MCl595

Thus by subtraction we find that

$$V' - V = \left[\frac{kT}{e} \right] \ln \left[\frac{I'}{I} \right]$$

Let $I' = i_o + i_y = i_o (1+y)$, and let $I = i_o - i_y = i_o (1-y)$

$$\text{then } V_y = V' - V = \left[\frac{kT}{e} \right] \ln \left[\frac{1+y}{1-y} \right]$$

Substituting this value of V_y into equation (1) we find

$$z = \ln \left[\frac{1+y}{1-y} \right] = \ln(z)$$

and noting also that $e^{\ln(z)} = z$ we find that $f(V_y)$ is given by

$$f(V_y) = \frac{\frac{1-z}{z}}{(1+z)(1+\frac{1}{z})} = \frac{(1-z)}{(1+z)}$$

Substituting for z we find that $f(V_y) = -y = \frac{-V_y}{i_o R_y}$

Therefore we find that for the MC1595 the output current I_d is given by

$$I_d \text{ is given by } I_d = \frac{-2V_x V_y}{i_o R_x R_y}$$

We have assumed that the diodes and transistors have matched characteristics, if this is not true we expect that the output will contain non-linear terms.