TECHNICAL INSTRUCTION
GP.3
DIGITAL METHODS OF CONTROLLING FREQUENCY AND PHASE

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DIGITAL METHODS OF CONTROLLING FREQUENCY AND PHASE

Frequency Dividers With Fractional Division Ratios

The use of bistable multivibrator counting circuits is described in Television Engineering, Vol.4. The division ratio of such circuits is always a whole number. Frequency translation by means of a modulator can be used to obtain a division ratio which is a fraction and not a whole number. Such a modulator is usually followed by a band-pass filter, to select the required sum or difference frequency, and by a Schmitt trigger circuit to reshape the output waveform of the filter.

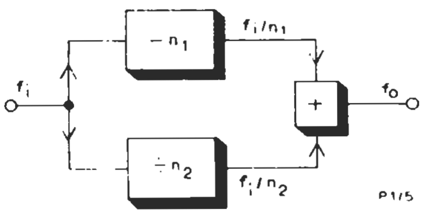

Fig.1. Frequency Divider with Fractional Division Ratio.

A block diagram of such a divider is given in Fig.1. In this Instruction the symbol with the + sign represents a combination of modulator, band-pass filter tuned to the *sum* frequency and a Schmitt trigger circuit. If the filter is tuned to the difference frequency, the symbol contains the – sign. The output frequency of the divider (f_o) is given by:

$$f_o = f_i/n_1 + f_i/n_2 = f_i \frac{n_1 + n_2}{n_1 . n_2}$$

The division ratio is given by:

$$f_i/f_o = \frac{n_1 . n_2}{n_1 + n_2} \quad \ldots\ldots \quad (1)$$

More complicated circuits involving more counters and modulators can be used.

Digital Phase Shifting

Each time a counter circuit produces an output pulse, it is said to have completed a count. By changing the division ratio for just one count the output pulse can be advanced or retarded relative to the input waveform. This change of timing of the output pulse can also be regarded as a phase shift and the change in timing, as a fraction of the output pulse period, can be expressed in degrees.

The waveforms in Fig. 2 illustrate the change produced by altering the division ratio for one count only. Fig.2(a) shows the input waveform of a divide-by-5 counter and Fig.2(b) shows its normal

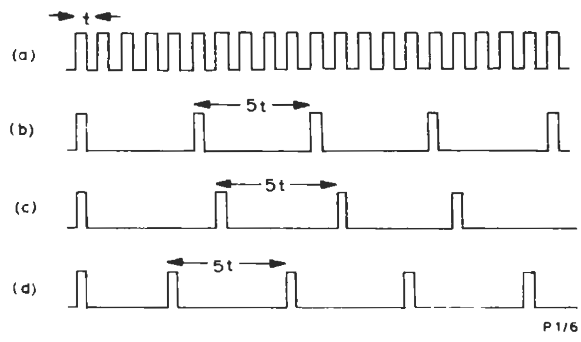

Fig.2. Waveforms Illustrating Digital Phase Shifting.

output waveform. In Fig.2(c) the division ratio is changed to divide-by-6 for the first count and in Fig.2(d) the ratio is changed to divide-by-4 for the first count.

In both these instances, the normal division ratio is maintained after the altered count. The change in timing in these examples is the period of one input pulse, *t*, and the phase shift is the ratio of this time to the normal period of the output pulse. The phase shift expressed in degrees is, therefore, 360/5: i.e. 72 degrees.

When the frequency is changed, either by dividing or modulating, the change in timing or the phase-shift (but not both) is altered. These effects are explained in detail below.

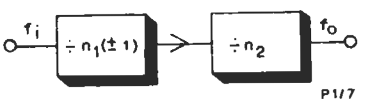

Fig.3. Counter Arrangement used to Illustrate Effect of Change of Input Timing and Phase.

Effects On Timing and Phase Shifts

Counter Circuits

The block diagram shown in Fig.3 is used to illustrate the manner in which a counter affects a timing change and a phase shift in its input waveform. This timing change and phase shift are considered as being produced by a single miscount in the first counter circuit in the manner described above.

In this instance the normal output of the first counter circuit has a frequency of f_i/n_1. The period t_1 of this waveform is given by:

$$t_1 = n_1/f_i$$

If the division ratio of the first counter is changed by one the period t'_1 is given by:

$$t'_1 = \frac{n_1 \pm 1}{f_i}$$

The difference between these two periods is clearly the period of the input waveform, $1/f_i$.

The phase shift in the output waveform of the first counter is given by:

$$\frac{t_1 \sim t'}{t_1} \times 360^\circ = (1/f_i \div n_1/f_i) \times 360^\circ$$

$$= \frac{360^\circ}{n_1} \quad \ldots\ldots \quad (2)$$

Consider the second counter. For each pulse which appears at the output of the counter, n_2 pulses are required at the input. If the period between two of these input pulses is changed for any reason, the output period of the counter is changed by the same amount of time.

Thus a change in timing of the input waveform is unaffected by a counter circuit.

The normal period of the input waveform of the second counter t_1 is given by:

$$t_1 = n_1/f_i$$

The normal period of the output waveform t_2 is given by:

$$t_2 = (n_1 \times n_2)/f_i$$

The time change $1/f_i$ causes a phase shift in the input waveform a_i given by (see equation 2):

$$a_i = (1/f_1 \div n_i/f_i) \times 360^\circ$$

$$= \frac{360^\circ}{n_1}$$

The corresponding phase shift in the output waveform a_0 is given by:

$$a_o = (1/f_1 \div n_1 n_2/f_1) \times 360^\circ$$

$$= \frac{360^\circ}{n_1 n_2} \quad \ldots\ldots \quad (3)$$

Thus phase shift at the output of a counter, relative to the phase shift at its input, is reduced by the division ratio.

Modulators

Consider a modulator whose inputs have pulse repetition frequencies of f_1 and f_2. These input signals can be represented by their fundamental components

$$\text{Sin } 2\pi f_1 t$$

$$\text{and Sin } (2\pi f_2 t + a)$$

where a is the relative phase difference between the signals at time t = o.

All modulation processes of the type considered here produce terms which are the product of the input signals. These product terms may be expressed in terms of the sum and difference frequencies which shows how a modulator can be used as a frequency adder.

The output of the modulator is given by the product:

$$\text{Sin } 2\pi f_1 t \times \text{Sin } (2\pi f_2 t + a)$$

$$= \text{Cos } (2\pi \overline{f_2 - f_1}\, t + a) - \text{Cos } (2\pi \overline{f_2 + f_1}\, t + a)$$

Thus the phase shift a as an angle is not changed by the modulation process. The change in timing varies inversely as the change of frequency through the modulator. This is given by the relationship between a timing change t, the frequency f and a phase shift a:

$$t = 1/f \times a/360^\circ \quad \ldots \quad \ldots \quad (4)$$

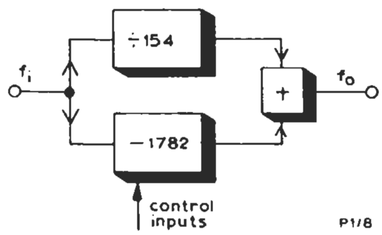

Fig.4. Fractional Divider from GE1/520.

Examples of Digital Control

(a) In this example, shown in Fig.4 and taken from a Drive Unit GE1/520, a fractional division ratio is required. Substituting in equation (1), the division ratio is given by:

$$\frac{f_1}{f_o} = \frac{154 \times 1782}{154 + 1782}$$

$$= 141 \cdot 75$$

The timing change produced by altering the division ratio of the divide-by-1782 counter by one (to either divide-by-1781 or divide-by-1783) for one count is

$$= 1/f_i$$

This can be expressed as a phase shift by substituting in equation (2):

$$= 360^\circ/1782$$

$$\approx 0{\cdot}2^\circ$$

The phase shift produced in the output signal is also approximately 0·2 degrees, but the change in timing of the output signal is given by substituting in equation (4):

$$t = 32 \times 10^{-6} \times \frac{0{\cdot}2}{360}$$

$$= \frac{32 \times 10^{-6}}{1782}$$

$$\approx 18\text{ns.}$$

(b) In this example, shown in Fig.5 and taken from a Colour Sub-carrier Phase Shifter EP1L/509, the output frequency is the same as the input frequency. This can be shown as follows:

$$f_a = f_1/16$$

$$f_b = f_1 - f_a$$
$$= \frac{15}{16} f_1$$

$$f_c = f_b/15$$
$$= f_i/16$$

$$f_d = f_b + f_c$$
$$= \frac{15}{16} f_i + \frac{1}{16} f_i$$
$$= f_i$$

$$f_e = f_d/16$$
$$= f_i/16$$

$$f_o = f_e + f_b$$
$$= \frac{1}{16} f_i + \frac{15}{16} f_i.$$
$$= f_i$$

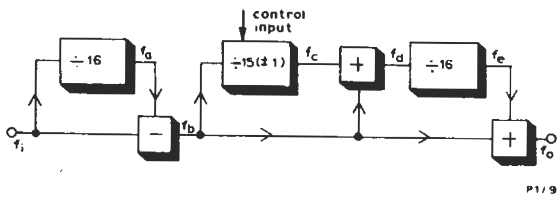

Fig.5. Digital Phase-shifter from EP1L/509.

The phase shift is produced by changing the division ratio of the divide-by-15 counter to either divide-by-14 or divide-by-16.
The phase shift in the signal out of the divide-by-15 counter due to a miscount is given by substituting in equation (2):

$$a_c = 360^\circ/15$$
$$= 24^\circ$$

This phase shift is unaffected by the modulator and so the phase shift a_d is given by:

$$a_d = a_c = 24^\circ$$

The phase shift is reduced in the following divide-by-16 counter to give:

$$a_e = 24^\circ/16$$
$$1.5^\circ$$

which is also the phase shift produced in the output signal by one miscount in the divide-by-15 counter.
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